Module 3

Mathematics 6

Patterns

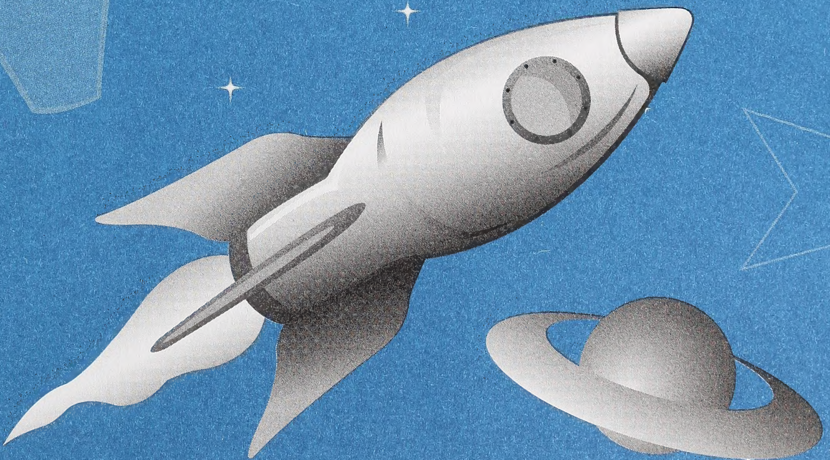
Learning Technologies Branch

Alberta
LEARNING

Mathematics 6

Module 3

Patterns



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Mathematics 6
Module 3: Patterns
Student Module Booklet
Learning Technologies Branch
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The Learning Technologies Branch acknowledges with appreciation the Alberta Distance Learning Centre and Pembina Hills Regional Division No. 7 for their review of this Student Module Booklet.

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome to
Mathematics 6

Mathematics 6 contains nine modules.
You should work through the modules in order (from 1 to 9) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Module 1
Estimating and Representing Number

Module 2
Number Operations

Module 3
Patterns

Module 4
Fractions, Ratio, and Percent

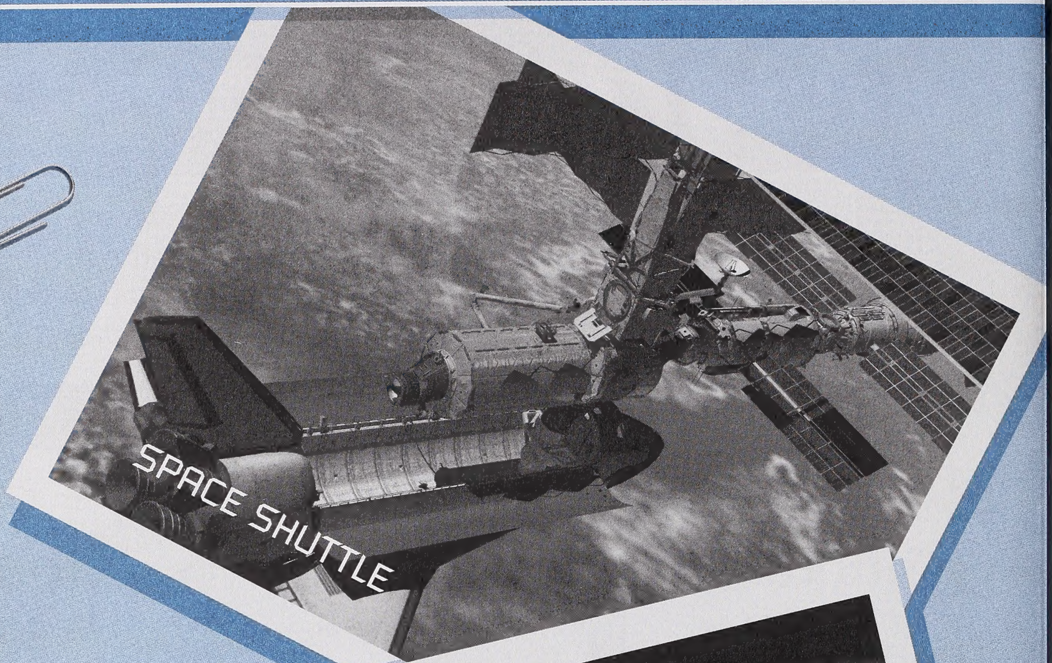
Module 5
Measurement

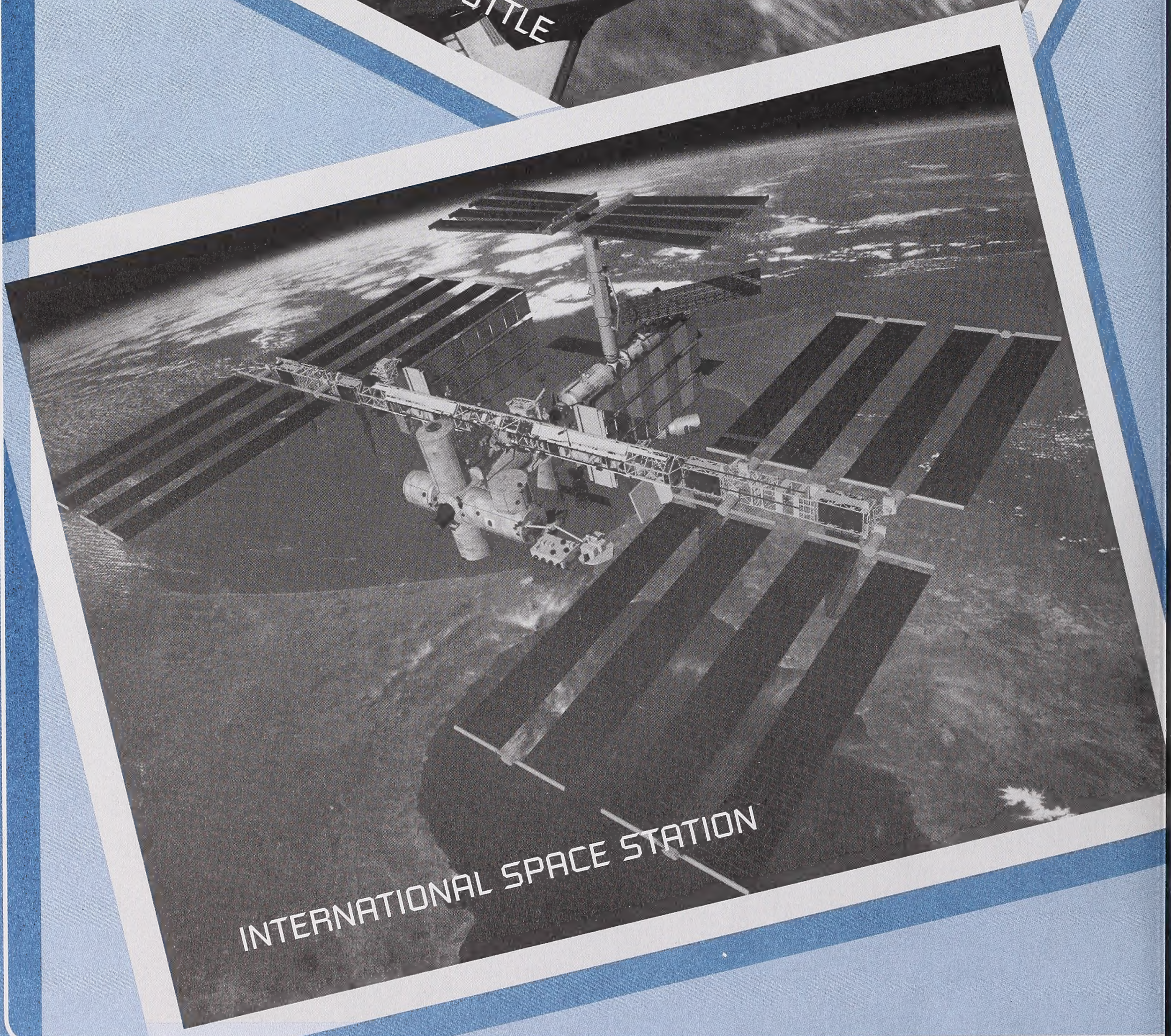
Module 6
Angles, Shapes, and Objects

Module 7
Transformations

Module 8
Data Analysis

Module 9
Probability


SPACE SHUTTLE


INTERNATIONAL SPACE STATION

Adventures in Outer Space

Matthew: Wow, what a wonderful experience it was meeting Colonel Chris Hadfield at the Odyssium! He gave a presentation here in Edmonton on July 9, 2001, and talked about his adventures in space, including his mission aboard the Space Shuttle *Endeavor* to attach Canadarm2 to the International Space Station.

It's too bad you missed it, Kylee. You were away visiting your grandmother in Slave Lake.

Kylee: My trip was great, but I sure wish I could have heard Colonel Hadfield talk about being the first Canadian to walk in space. But I've got great news for you, Matthew! Commander Claire from the International Space Station is coming to town, and you and I will be spending some time with her.

I can't wait to hear about her adventures in space!

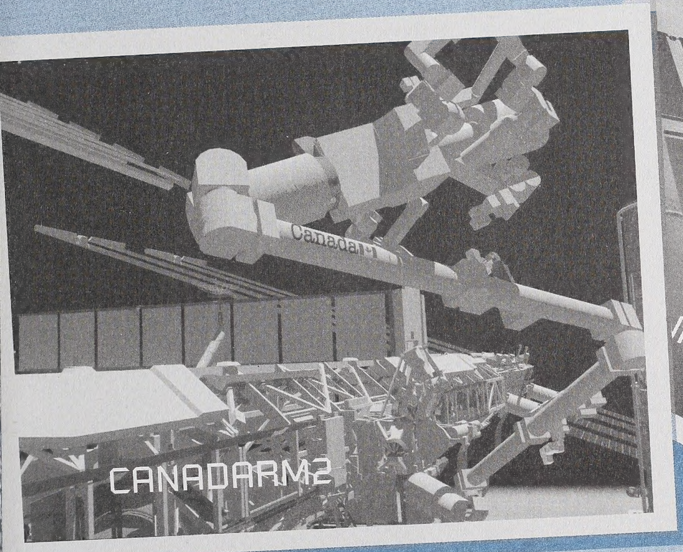


Table of Contents


Commander Claire

Course Features

Take the time to look through the Student Module Booklets and the Assignment Booklets and notice the following design features:

- Each module has a Module Overview, Module Summary, and Review.
- Each module has several lessons. Each lesson focuses on a big idea that is central to the topic being learned in the module.
- Each lesson has several activities. An activity in each lesson is related to the Adventures in Outer Space theme.
- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.
- Each module has special exercises that focus on certain mathematical skills. The Numbers in the News project involves a scavenger hunt for samples of math in everyday life. The Keystrokes exercise introduces some “funky features” of the calculator that can be used to explore and practise important number ideas. Just the Facts gives you the opportunity to practise your basic number facts by doing a timed drill with your home instructor. The Mental Math exercise introduces an estimation skill or mental-computation strategy that you can use to sharpen your mental math skills.
- Each module references the Mathematics 6 Companion CD that includes additional material for review and mastery.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

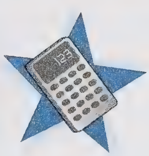
In order to complete the course, you will need a copy of the Mathematics 6 textbook, *Quest 2000: Exploring Mathematics*, *Grade 6*, the soft-cover book *Quest 2000: Exploring Mathematics: Practice and Homework Book*, *Grade 6*, a basic four-operation calculator (such as the TI-108 calculator), and various manipulatives (base ten blocks and pattern blocks).

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

You will also need access to a computer to view material on the Mathematics 6 Companion CD.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.



Use your calculator.

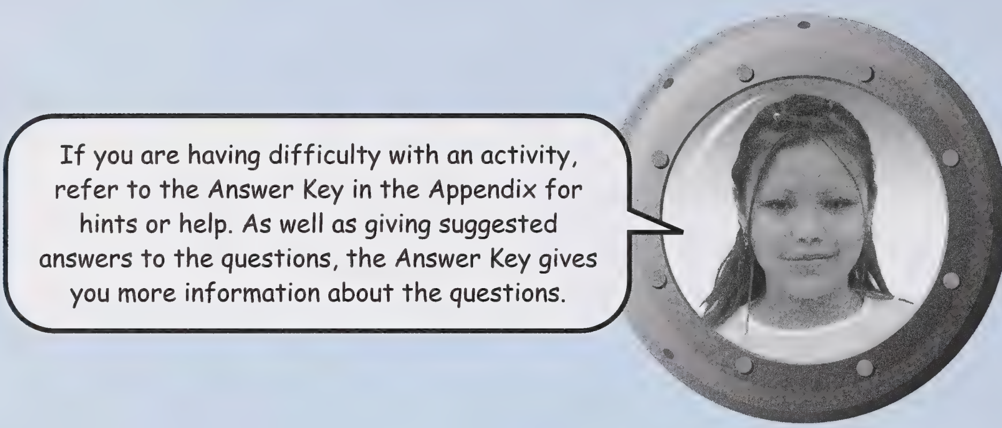
Use the Internet.

Refer to the textbook or the Practice and Homework Book.

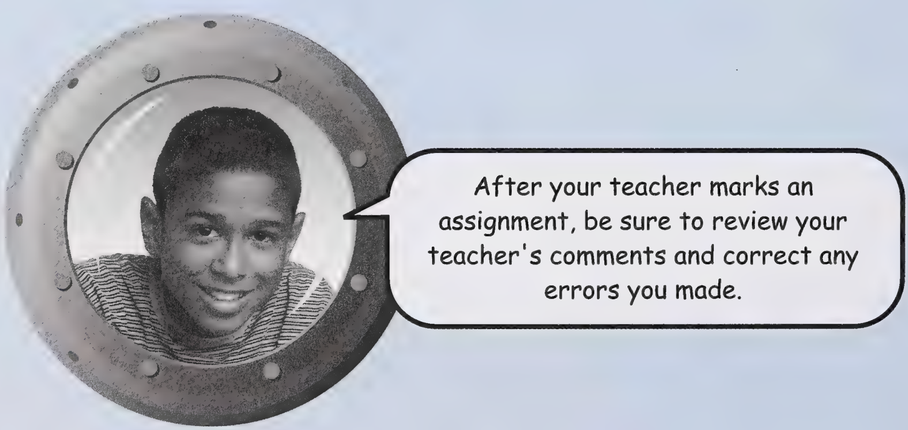
Use the Mathematics 6 Companion CD.

Assessment and Feedback

The Mathematics 6 course is carefully designed to give you many opportunities to discover how well you are doing. In every activity you will be asked to turn to the Appendix to check your answers. Completing the activities and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.



Twice in each module you will be asked to give your teacher your completed assignments to mark. Your teacher will give you feedback on how you are doing.



There will be a final test at the end of the course. You can prepare for the final test by completing the Review at the end of each module.

Module Overview

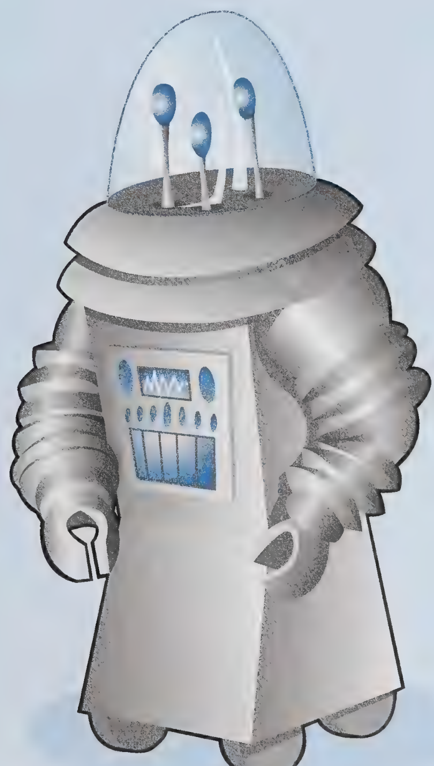
Ancient seafarers used the sun, the moon, and the stars to guide their journeys. Perhaps, when modern explorers journey into space beyond the solar system, they will also use sky charts to locate their position. These charts will likely show familiar patterns of stars or constellations. Did you know that many of these constellations were first named 5000 years ago by the peoples of ancient Mesopotamia? Can you locate the Big Dipper, Orion, Cassiopeia, or other constellations in the night sky?

Many people think of mathematics as a search for patterns. In this module you will explore number patterns, describe them, and represent them using graphs. You will use these number patterns to solve a variety of problems.

Lesson 1
Whole Number Patterns

Lesson 2
Representing Patterns

Lesson 3
Beginning Algebra



Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 3A

Lesson 1 Assignment	30 marks
Lesson 2 Assignment	30 marks

Assignment Booklet 3B

Lesson 3 Assignment	30 marks
Numbers in the News	10 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 3A to your teacher before you begin Lesson 3. You will submit Assignment Booklet 3B to your teacher at the end of this module.

Numbers in the News

Read through the following list before you begin Module 3. Begin by collecting samples of the ideas you already understand; others you may collect as you learn about them in the module. The samples you collect will depend on the newspapers or magazines you use.

Scavenger Hunt

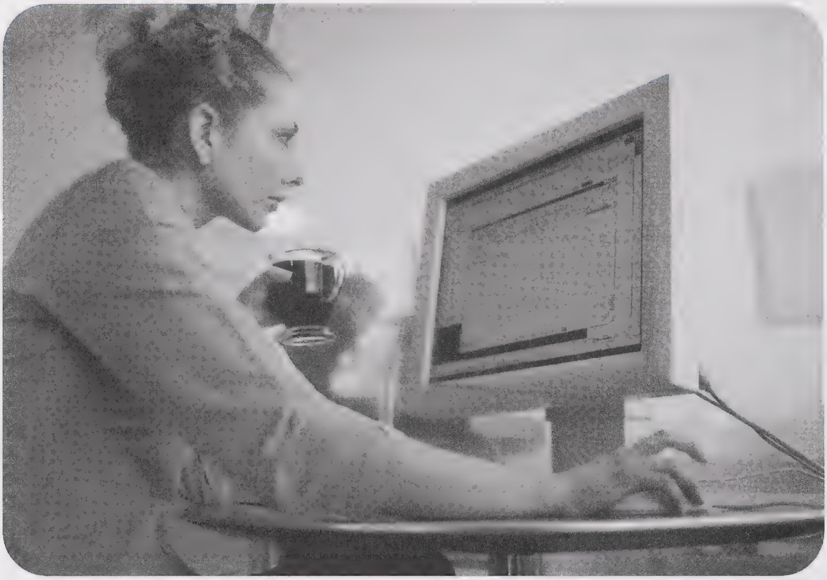
Cut out articles or advertisements from newspapers or magazines that show patterns being used in different situations. Here are some suggestions of things to look for:

- pictures of geometric shapes that form a repeating or growing pattern
- number patterns
- situations that use multiples, factors, prime numbers, or composite numbers
- lists or tables that show relationships between numbers
- graphs that show how numbers are related
- pictures that show objects or numbers being balanced

You will find further instructions for completing and submitting your project in Assignment Booklet 3B.

Lesson 1

Whole Number Patterns



When people shop or sell items over the Internet, they want to be sure that their transactions are safe and that other individuals can't intercept their orders or access their accounts. Computer security involves coding messages so that others can't understand them. The preparation and study of coded messages, called *cryptography*, involves the use of **prime numbers**. Prime numbers, such as 2, 3, 5, and 7, are whole numbers larger than 1 that can be divided only by themselves and 1.

In this lesson you will explore an ancient method for finding prime numbers and learn about prime factorization. As well, you will extend your understanding of factors and multiples to include **common factors**, **common multiples**, the **greatest common factor**, and the **least common multiple**. You will use these ideas to solve problems.

Activity 1

Today you will explore least common multiples.

Eating in space has improved tremendously since early astronauts dined on freeze-dried cubes and semi-liquid food from a tube!

If you have access to the Internet, you can find out more about mission menus at the following website:

http://spaceflight.nasa.gov

Before the launch of a mission to the International Space Station, each team of astronauts plans a menu for a particular number of days. The menu repeats throughout the mission.

For a particular mission, Team A planned a menu that repeated every six days and Team B planned a menu that repeated every eight days. The mission ended on the day that both teams completed a menu cycle together for the first time. To find how many days the mission lasted, use counters to model both sets of multiples on a hundred chart. (Remove a hundred chart from the Appendix.)

1. Beginning on the number 6, place a counter on every sixth number on the hundred chart to represent the days that Team A completed a menu cycle. As soon as you see a pattern starting to appear, complete the chart by predicting the rest of the days that Team A completed a menu cycle.

Use your calculator to verify your pattern.

Keystrokes	ON/C	6	+	=	=	=	···
Display	0	6	6	6	12	18	···

Record your results on your hundred chart. Draw a circle around each multiple of 6 as you remove the counters.

2. Beginning with the number 8, place a counter on every eighth number on the hundred chart to represent the days that Team B completed a menu cycle. As soon as you see a pattern starting to appear, complete the chart by predicting the rest of the days that Team B completed a menu cycle.

Use your calculator to verify your pattern.

Keystrokes	ON/C	8	+	=	=	=	···
Display	0	8	8	8	16	24	···

Draw a triangle around each multiple of 8 as you remove the counters.

Check your answers on page 87 in the Appendix.

3. Numbers on the hundred chart that have both circles and triangles around them are common multiples of 6 and 8. List those that you found. Describe the pattern.

4. The first number that has both a circle and a triangle around it is called the **least common multiple** (LCM) of 6 and 8. What is the LCM of 6 and 8?

5. How are the multiples of 6 and the multiples of 8 related to the list of their common multiples?

6. The menu cycles of both teams of astronauts began on the first day of their mission. After how many days did the two teams complete a menu cycle together?

Check your answers on page 87 in the Appendix.

You'll need to look at a calendar to complete questions 7, 8, and 9.

7. The first day of the mission was September 2. September 2 was also the first day of each menu cycle. Keep listing the dates on which the teams completed their separate menu cycles until you reach the date when the two teams first completed a menu cycle together. On what date did that happen?

8. The missions for Team A and Team B were extended until December. How many times did the two teams complete a menu cycle together before December? On what dates did that happen?

NASA

Astronauts Brian Duffy (left) and William S. McArthur, Jr., sample food during a crew menu-evaluation session.

9. Team C, a third team on the same mission, completed a menu cycle every twelve days.

   a. List the dates before December on which Team C completed its menu cycles. Circle the dates that all three teams completed a menu cycle together.

   b. After how many days did the three teams complete a menu cycle together?

   c. What was the date of the first day all three teams completed a menu cycle together?

   d. Explain why the menu cycle of Team C overlapped so well with those of the other two teams.

   e. Explain how you could use a hundred chart and counters to show the LCM of 6, 8, and 12.

Check your answers on pages 87 and 88 in the Appendix.

In this activity you examined a method of finding common multiples and least common multiples.

Example

List the multiples of 6 and 4. Which are the common multiples? What is the least common multiple?

You can list multiples by skip counting.

+4 +4 +4 +4 +4 +4 +4 +4

4, 8, 12, 16, 20, 24, 28, 32, 36, …

6, 12, 18, 24, 30, 36, 42, …

+6 +6 +6 +6 +6 +6

Common multiples of 4 and 6 are circled. These are 12, 24, 36, … . Do you see a way of extending the list of common multiples? Simply list the mulitiples of 12. The least common multiple is 12.

Activity 2

Today you will explore common factors and greatest common factors.

Remember, a factor is a whole number that evenly divides a given whole number, or factors are numbers multiplied together to give a product.

Example

List the factors of 6.

Since $6 = 1 \times 6$ and $6 = 2 \times 3$, the numbers 1, 2, 3, and 6 are all factors of 6.

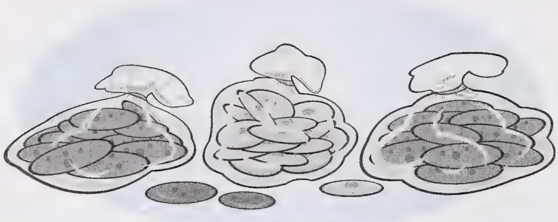
Philip baked 24 chocolate chip cookies and Helen baked 30 shortbread cookies for a bake sale. They want to put their cookies in bags so that all bags contain the same number of cookies, without having any cookies left over and without mixing the cookies. Use counters or tiles to explore the different ways of solving this problem.

1. a. Use 24 counters to find all of the different ways Philip can separate his cookies into groups with the same number of cookies in each group. Draw and label diagrams to show your answer.

   b. Use 30 counters to find all of the different ways Helen can separate her cookies into groups with the same number of cookies in each group. Draw and label diagrams to show your answer.

2. How many cookies could Philip and Helen put in each bag?

3. What is the largest group of cookies that can be made with both 24 and 30 cookies?



Check your answers on pages 89 and 90 in the Appendix.

4. List all the factors of 24 and 30. Circle the numbers that appear in both lists (common factors). Draw a box around the greatest common factor (GCF).

5. While visualizing what you did with the counters in question 1, describe a way to find the GCF of any two numbers without using counters.

6. Use your method from question 5 to answer the following questions.

   a. Find all the common factors of 20 and 36.
   b. What is the GCF of 20 and 36?

7. Can you find a pair of numbers that has no common factor at all? Explain your thinking.

8. Neil made 18 oatmeal cookies for the same bake sale as Philip and Helen.

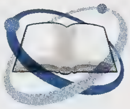
a. If Philip, Helen, and Neil wanted to make packages of cookies that are all equal in size, how many cookies can they put in each package? Explain.

b. What is the largest package of cookies that they all can make?

Check your answers on pages 90 and 91 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.



Turn to page 22 of the Practice and Homework Book and complete questions 1, 2, 3, 6, 7, and 8.

Questions 4, 5, 9, 10, 13, and 14 are optional.

Discuss your answers with your home instructor.

Activity 3

Today you will examine prime numbers.

One of the greatest ancient mathematicians was Eratosthenes of Cyrene (276–194 B.C.). He was born in Cyrene, in what is present-day Libya, and later he became director of the Great Library in Alexandria, Egypt. It was there that he made most of his contributions to mathematics, astronomy, and other sciences.

Eratosthenes calculated the circumference of Earth, determined the tilt of Earth's axes, drew star maps, and developed a method for finding prime numbers.

WWW

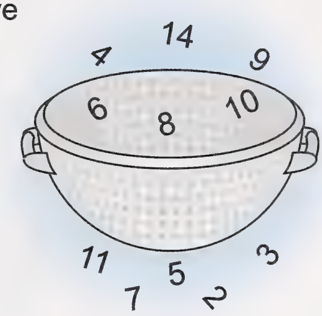
If you have access to the Internet, you can find out more about Eratosthenes and prime numbers at the following website:

http://turnbull.dcs.st-and.ac.uk/history/Mathematics/Eratosthenes.html

A prime number is a number that has exactly two factors: itself and 1. A **composite number** is a number that has more than two factors.

Unfortunately, given any prime number, there is no way to predict what the next one will be. Eratosthenes invented a method for finding prime numbers. This method is called the Sieve of Eratosthenes because it "sifts out" the composite numbers.

In this activity you will learn how and why the sieve works as you find all the prime numbers less than 100. Begin by taking out a copy of the hundred chart from the Appendix and some counters.



When you find a prime number, you will circle it on the hundred chart. You will "sift out" numbers that are not prime (all the multiples of prime numbers) by crossing them out on the chart.

1. Begin by crossing out 1 on the chart. Explain why 1 is not a prime number.

2. a. The next number on the chart is 2. Explain why 2 is a prime number. Draw a circle around 2 on the chart.

   b. What do you call numbers that are divisible by 2?

   c. Can any number that is divisible by 2 (any number that is a multiple of 2) be a prime number? Explain.

   d. Find all the multiples of 2 up to 100. Beginning on 4, skip count by 2s and place a counter on each number you say. As soon as you see a pattern starting to appear, finish placing the counters by predicting the remaining multiples of 2.

   Use the constant function on your calculator to verify your pattern.

Keystrokes	ON/C	2	+	2	=	=	⋯
Display	0	2	2	2	4	6	⋯

   Record your work on the hundred chart. Cross out each multiple of 2 as you remove the counters.

3. a. The next number on the hundred chart is 3. Explain why 3 is a prime number. Draw a circle around 3 on the chart.

b. Find all the multiples of 3. Beginning on 6, skip count by 3s and place a counter on each number you say. As soon as you see a pattern starting to appear, finish placing the counters by predicting the remaining multiples of 3.

Use the constant function on your calculator to verify your pattern.

Keystrokes	ON/C	3	+	3	=	=	···
Display	0	3	3	3	6	9	···

Record your work on the hundred chart. Cross out each multiple of 3 as you remove the counters.

c. Explain why some of the multiples of 3 were already crossed out on the chart.

4. a. The next number on the hundred chart that has not been crossed out is 5. Explain why 5 is a prime number. Draw a circle around 5 on the chart.

b. Find all the multiples of 5. Beginning on 10, skip count by 5s and place a counter on each number you say. As soon as you see a pattern starting to appear, finish placing the counters by predicting the remaining multiples of 5.

Use the constant function on your calculator to verify your pattern.

Keystrokes	ON/C	5	+	5	=	=	···
Display	0	5	5	5	10	15	···

Record your work on the hundred chart. Cross out each multiple of 5 as you remove the counters.

c. Explain why some of the multiples of 5 were already crossed out on the chart.

5. a. The next number on the chart that has not been crossed out is 7. Explain why 7 is a prime number. Draw a circle around 7 on the chart.

b. Find all the multiples of 7. Beginning on 14, skip count by 7s and place a counter on each number you say. As soon as you see a pattern starting to appear, finish placing the counters by predicting the remaining multiples of 7.

Use the constant function on your calculator to verify your pattern.

Keystrokes	ON/C	7	+	7	=	=	···
Display	0	7	7	7	14	21	···

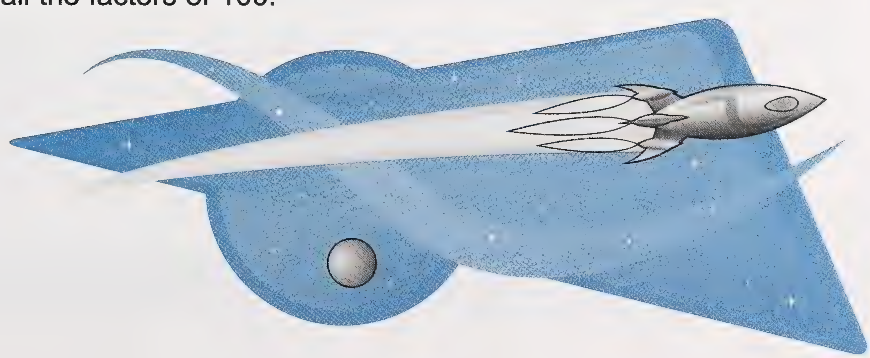
Record your work on the chart. Cross out each multiple of 7 as you remove the counters.

c. Explain why some of the multiples of 7 were already crossed out on the chart.

Check your answers on pages 91 to 93 in the Appendix.

Guess what? You've done it! You have just found all the prime numbers less than 100. Isn't it amazing that to find all the prime numbers less than 100, all you had to do was cross out multiples of just the first four prime numbers: 2, 3, 5, and 7!

It is important for you to understand why you do not have to test any more numbers. What happens is similar to what happens when you are asked to find all the factors of 100.

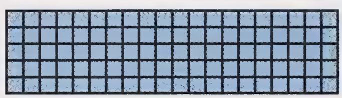


Recall that different rectangular arrays can be used to show all the factors of a number. All nine factors of 100 (1, 2, 4, 5, 10, 20, 25, 50, 100) can be shown with five rectangles.

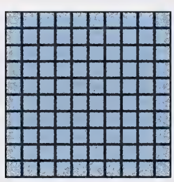
1×100 The diagram for 1×100 is not drawn, but imagine how long and narrow it would be.

2×50

4×25



5×20



10×10

6. Why don't you need to draw any more rectangles after the 10-by-10 square?

7. If one factor in a factor pair for 100 is greater than 10, what do you know about the other factor in the pair?

8. Explain why you can stop testing for prime numbers (and crossing out their multiples) before you reach 11 when finding all the prime numbers less than 100.

9. List the prime numbers less than 100.

Check your answers on page 94 in the Appendix.

Activity 4

A ring toss has been set up at the Cochrane Community Carnival. It has four red pylons, four white pylons, and four green pylons. Each game costs $0.25 and gives you four rings to toss.

If the ring you toss lands around a pylon, you earn a token that matches the colour of that pylon. The tokens have the following values:

- Red 2 points
- White 3 points
- Green 5 points

Your final score is found by multiplying the values of the tokens you earned.

1. Lyall ringed one red pylon and one green pylon. What was his final score? Show your work.

2. Aaron ringed one red pylon, two white pylons, and one green pylon. What was his final score? Show your work.

3. Chelsea ringed one pylon of each colour. What was her final score? Show your work.

Check your answers on page 94 in the Appendix.

4. a. What is the greatest final score you could get if you ring three pylons? Explain.

   b. What is the least final score you could get if you ring three pylons? Explain.

5. a. What is the greatest final score you could get if you ring four pylons? Explain.

   b. What is the least final score you could get if you ring four pylons? Explain.

6. Benita's final score was 20. When Reg asked her which tokens she had earned, she couldn't remember, but Reg said he could figure it out.

   a. Show how Reg could find which tokens Benita earned.

   b. Could Benita have earned different tokens to get the same score? Explain.

7. a. Reg said he could find which tokens anyone had earned if he knew their final score. Use the method you used for question 6 to find the point values of the tokens the following people earned. Find more than one answer, if possible. Write the values of the tokens in order from least to greatest.

Name	Final Score	Values of Tokens (from least to greatest)
Tyler	75	
Damon	24	
Vita	45	
Saul	54	
Cora	90	

b. Will a score made from ringing four pylons always be greater than a score made from ringing three pylons?

8. Were you able to find more than one set of tokens for any person in question 7?

Check your answers on pages 94 and 95 in the Appendix.

You have just discovered the **Fundamental Theorem of Arithmetic**!

Note: In this case, *unique* means that there is just one set of prime factors for any whole number.

Any whole number can be written as the product of a **unique** set of prime numbers.

The unique product of prime factors is called the **prime factorization** of that number.

For example, $12 = 2 \times 2 \times 3$. There is no other way of writing 12 as a product of prime numbers.

There are different ways to find the prime factorization of a number. You can use a factor tree or you can use repeated division by primes. Regardless of the method you use, it is customary to have the prime factors written from least to greatest in your final answer.

Example

Find the prime factorization of 24.

Using factor trees, it is possible to begin with different pairs of factors, but the prime factorization of 24 always ends with three 2s and one 3 being multiplied at the bottom of the tree.

$$\begin{array}{c} 24 \\ 4 \times 6 \\ 2 \times 2 \times 2 \times 3 \end{array}$$

$24 = 2 \times 2 \times 2 \times 3$

or

$$\begin{array}{c} 24 \\ 3 \times 8 \\ 3 \times 2 \times 4 \\ 3 \times 2 \times 2 \times 2 \end{array}$$

$24 = 2 \times 2 \times 2 \times 3$

or

$$\begin{array}{c} 24 \\ 2 \times 12 \\ 2 \times 2 \times 6 \\ 2 \times 2 \times 2 \times 3 \end{array}$$

$24 = 2 \times 2 \times 2 \times 3$

To factor 24, you can use repeated division by primes. Try dividing by 2 first.

$$\begin{array}{r} 2\,\underline{)\,24} \\ 12 \end{array} \longleftarrow 24 \div 2 = 12$$

Then divide 12 by 2.

$$\begin{array}{r} 2\,\underline{)\,24} \\ 2\,\underline{)\,12} \\ 6 \end{array} \longleftarrow 12 \div 2 = 6$$

Next, divide 6 by 2.

$2\overline{)24}$

$2\overline{)12}$

$2\overline{)6}$ $\longleftarrow$ $6 \div 2 = 3$

$\quad 3$

Finally, divide by 3.

$2\overline{)24}$

$2\overline{)12}$

$2\overline{)6}$

$3\overline{)3}$ $\longleftarrow$ $3 \div 3 = 1$

$\quad 1$

The numbers you divided by are the factors of 24.

$$24 = 2 \times 2 \times 2 \times 3$$

Using repeated division by primes, you can divide by the prime factors in different orders; but, once again, the prime factorization of 24 always ends with three 2s and one 3 being multiplied.

$2\overline{)24}$		$3\overline{)24}$		$2\overline{)24}$
$2\overline{)12}$		$2\overline{)\ 8}$		$3\overline{)12}$
$2\overline{)\ 6}$	or	$2\overline{)\ 4}$	or	$2\overline{)\ 4}$
$3\overline{)\ 3}$		$2\overline{)\ 2}$		$2\overline{)\ 2}$
1		1		1

9. Use factor trees or repeated division of primes to find the prime factorization of the following numbers.

a. 42 **b.** 110
c. 120 **d.** 360

10. For each of the numbers in question 9, tell what new token value (if any) would have to be included in the ring-toss game to be able to get the final scores listed in question 9.

Check your answers on pages 96 and 97 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Turn to page 24 of the Practice and Homework Book and complete questions 1, 3, 5, 7, 9, 12, and 14.

Questions 2, 4, 6, 8, 11, and 13 are optional.

Discuss your answers with your home instructor.

Challenge Activity

Sid made identical subs for the lunch special in his submarine-sandwich shop. He had 36 slices of cheese, 48 slices of turkey, and 42 slices of tomato. He put the same number of slices of cheese, the same number of slices of turkey, and the same number of slices of tomato in each sandwich. He used all the fillings he had.

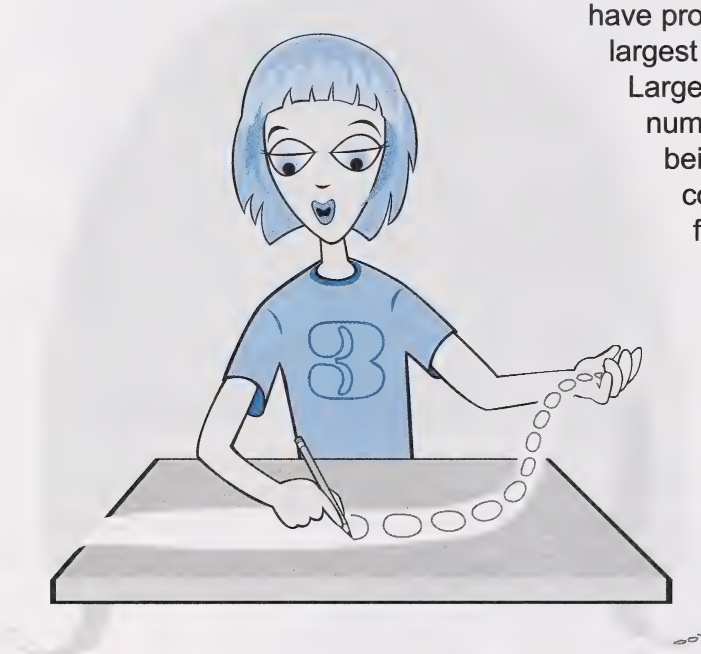
1. What is the greatest number of identical subs Sid could have made?

2. How many slices of each filling did Sid put in each sub?

Check your answers on page 97 in the Appendix.

Conclusion

In this lesson you extended your understanding of factors and multiples to include common factors, common multiples, the greatest common factor, and the least common multiple. You used the Sieve of Eratosthenes to find prime numbers, and you learned about prime factorization. You applied these ideas to solve problems.



Did you know that mathematicians have proven that there is no largest prime number? Larger and larger prime numbers are constantly being discovered using computers. In 1999, for example, mathematicians discovered a prime number that has 2 098 960 digits! If you wrote this prime number on a strip of paper with each digit taking up 0.5 cm, the strip of paper would be over 10 km long!

If you have access to the Internet, you can find out more about large prime numbers at the following website:

http://www.utm.edu/research/primes/

Turn to Assignment Booklet 3A and complete the Lesson 1 Assignment.

Keep Assignment Booklet 3A until you have completed the entire booklet.

Lesson 2

Representing Patterns

Mathematics occurs in the patterns you see all around you! For example, have you ever looked carefully at a floor that is tiled in a checkerboard pattern? In the one above, there is a single dark tile in the corner. Then, working down, there are three dark tiles, then five, then seven, and so on. The numbers 1, 3, 5, 7, …, are odd numbers!

To find how many dark tiles there are altogether, you could start adding the odd numbers as you work down from the corner. What is the pattern?

$1 = 1 \times 1$

$1 + 3 = 4 = 2 \times 2$

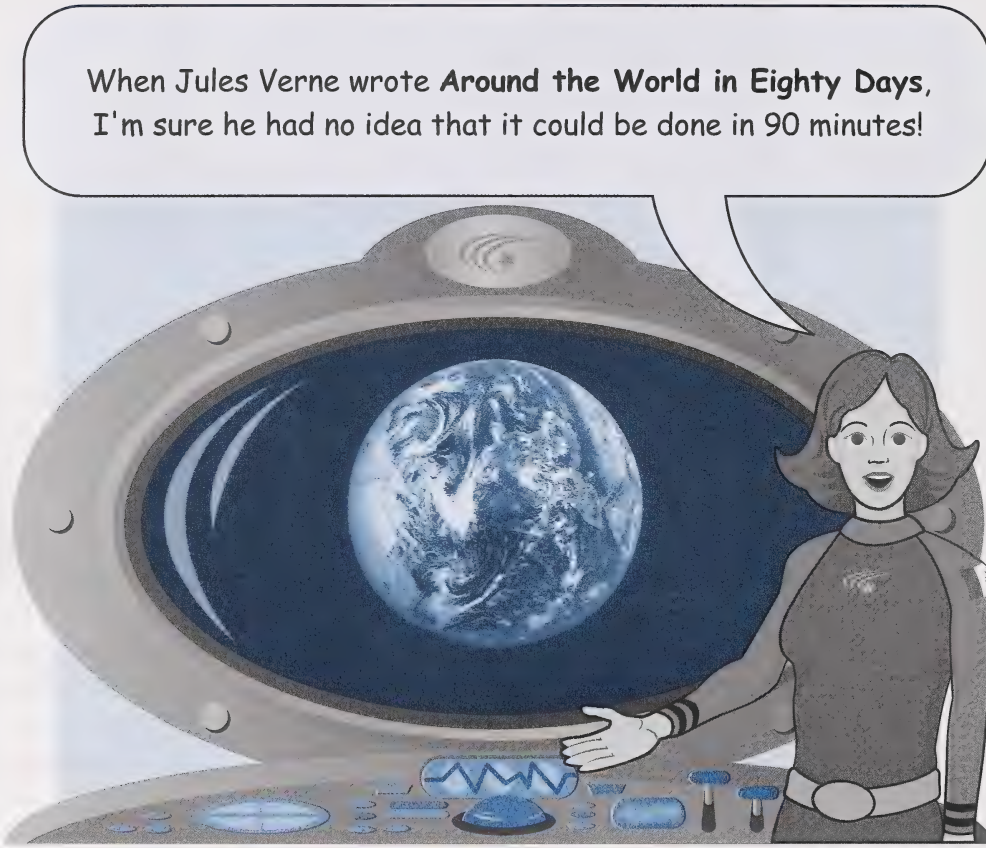
$1 + 3 + 5 = 9 = 3 \times 3$

$1 + 3 + 5 + 7 = 16 = 4 \times 4$

In this lesson you will explore a variety of patterns. You will describe, in words and mathematically, the relationships you see in picture and number patterns. You will use these relationships to extend these patterns. Finally, you will see how those patterns can be represented by graphs.

Activity 1

Today you will examine patterns involving speed, distance, and time.



The following table gives some information about the International Space Station as it orbits Earth. Use this information to answer questions 1 to 9.

Number of Orbits	1	2	3	4	5	6
Time (h)	1.5	3	4.5			
Distance Travelled (km)	43 000	86 000	129 000			

1. a. How long does it take, in hours, for the International Space Station to orbit once around Earth?

   b. Describe, in words, the relationship between the time, in hours, and the number of orbits.

2. a. What is the length, in kilometres, of one orbit around Earth?

   b. Describe, in words, the relationship between the total distance travelled and the number of orbits.

NASA

3. Copy and complete the table from the previous page.

4. How many orbits does the International Space Station make in one day? Explain.

5. Use your answer to question 4 to first estimate and then calculate (using your calculator) the distance in kilometres that the space station travels in one day. Explain.

Check your answers on pages 97 and 98 in the Appendix.

When astronauts in orbit travel around Earth, they pass through all the time zones every 90 minutes or so. So, how do they keep track of what time it is on the spacecraft? They use universal time co-ordinated (UTC).

Universal time co-ordinated is the way that time is presented in the International System of Units (SI). It is the time of day at the prime meridian (0° longitude), which passes through Greenwich, England. You may also see it expressed as Greenwich mean time (GMT) or Zulu time (Z time).

6. One crew of astronauts arrived at the International Space Station on October 15 at 07:00 UTC. They left on November 26 at 16:00 UTC.

a. How many orbits did the crew make while on the space station? Explain.

b. Use your answer to question 6.a. to first estimate and then calculate the total distance in kilometres the crew travelled while on the space station. Explain.

7. A space shuttle delivering supplies docked with the International Space Station at 13:30 UTC. The shuttle left the space station when it had just completed two orbits around Earth. What time did the shuttle leave the space station? Explain.

8. The cumulative crew time in orbit for three astronauts on another mission was 705 h. (Cumulative crew time means the total time for the three astronauts.)

a. Estimate and then calculate how many days the mission lasted.

b. Use your answer to question 8.a. to estimate and then calculate how many orbits each astronaut made.

Question 9 is optional.

9. How many times would the International Space Station orbit Earth in the same time it took Phileas Fogg to go around the world in his hot-air balloon (in Jules Verne's novel *Around the World in Eighty Days*)? Explain.

Check your answers on pages 98 to 100 in the Appendix.

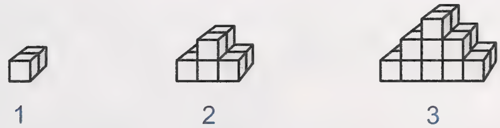
Activity 2

Today you will examine patterns in shapes built with cubes.

After reading about the strange pyramidal forms on Mars,
I had a fantastic dream . . .

NASA (Mars)

There appeared to be a pattern in the construction of the strange Martian buildings that Commander Claire saw arranged in a row in her dream. When she awoke, she was so excited that she built models of the buildings. The following pictures show her models of the first three buildings.



Use small base ten cubes or centicubes to build models of the first three buildings.

1. Describe how you could build a model of the fourth building.

2. Build a model of the fourth building and draw a picture of it.

3. Use your models to copy and complete a table like the following in your notebook.

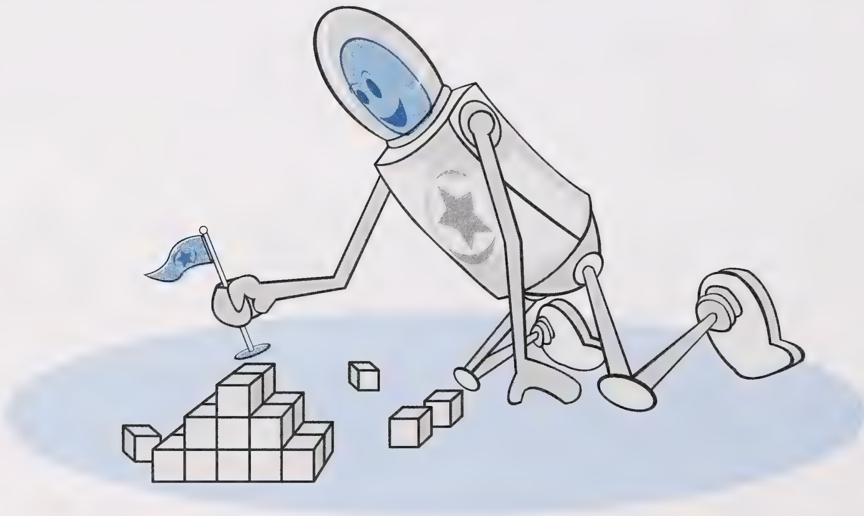
Model Number	Number of Layers	Number of Cubes in Bottom Layer	Total Number of Cubes in Model
1	1	2	2
2	2		
3			
4			

4. How is the number of layers related to the model number?

5. How many more blocks are in any given layer than the layer just above it?

6. What two-operation method can you use with the model number to get the number of cubes in the bottom layer?

Check your answers on pages 100 and 101 in the Appendix.



7. Nanook discovered a pattern using the model number to find the total number of cubes in the model. He multiplied the model number by itself, and then he doubled that number.

   a. Test Nanook's method by completing the first four rows of a table like the following. Check your answers with the table you completed in question 3.

Model Number	Number of Cubes in Bottom Layer	Total Number of Cubes in Model
1		
2		
3		
4		
5		
6		

   b. Use your answer from question 6 to complete the middle column of the table.

   c. Use Nanook's method to complete the right-hand column of the table.

8. Using the information from the table in question 7, complete a graph to show the relationship between the model number and the number of cubes needed to build it. Use a sheet of centimetre grid paper from the Appendix.

9. a. Use the graph you drew in question 8 to predict the number of cubes needed to build the seventh model. Explain.

   b. Verify your prediction by using Nanook's method from question 7.

Check your answers on page 101 in the Appendix.

Questions 10 and 11 are optional.

10. The more Claire examined her models, the more patterns she saw.

a. Use your models to complete a table like the following.

Model Number	Number of Squares on Front Face	Number of Horizontal Roof Squares
1	1	2
2	$1+3=4$	6
3	$1+3+5=9$	10
4		
5		
6		
7		

b. How is the number of squares on the front face related to the model number?

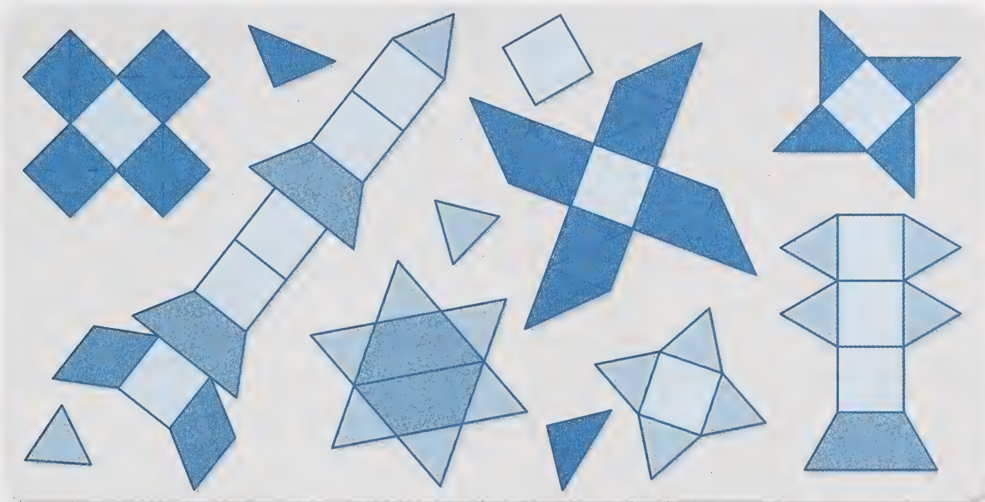
c. How is the number of horizontal roof squares related to the number of cubes in the bottom layer?

11. A model of the largest Martian building has a total of 144 squares on its front face. How many buildings were there in all?

Check your answers on page 102 in the Appendix.

Activity 3

Today you will graph mathematical relationships involving patterns.



You can use pattern blocks to build models and draw pictures that represent them. You can use words, T-tables, or graphs to describe the relationships among the blocks you use.

Turn to page 12 in your textbook. Use Figure 1, Figure 2, Figure 3, and the T-tables to answer questions 1 to 4.

1. a. Describe the ways in which Figure 1, Figure 2, and Figure 3 are all alike.

   b. Explain how Figure 2 is different from Figure 1.

   c. Explain how Figure 3 is different from Figure 2.

   d. What rule tells how the pattern grows (changes) from one figure to the next?

2. a. Copy and complete the following T-table in your notebook. The ordered pairs are formed using one number from each column as shown. Copy and complete the ordered pair list.

Figure Number	Number of Triangles in Each Arm	Ordered Pair
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)
4	4	(4, 4)
5	5	(5, 5)
6	6	(6, 6)
7		
8		

b. Graph the number of triangles in each arm. Use a sheet of graph paper from the Appendix.

c. What rule relates the figure number to the number of triangles in each arm?

3. a. Copy and complete a T-table and an ordered pair list like the following in your notebook.

Figure Number	Total Number of Triangles	Ordered Pair
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4		
5		
6		
7		
8		

b. Graph the total number of triangles. Use a sheet of graph paper from the Appendix.

c. What is the rule that relates the figure number to the total number of triangles?

Check your answers on pages 102 to 104 in the Appendix.

4. a. Copy and complete a T-table and an ordered pair list like the following in your notebook.

Figure Number	Total Number of Pieces	Ordered Pair
1	5	(1, 5)
2	9	(2, 9)
3	13	(3, 13)
4		
5		
6		
7		
8		

b. Graph the total number of pieces for each figure. Use a sheet of graph paper from the Appendix.

c. What rule using two operations can you use with the figure number to get the total number of pieces?

Check your answers on page 105 in the Appendix.

Turn to page 13 in your textbook. Use Figure 1, Figure 2, Figure 3, and the T-tables to answer questions 5 to 8.

5. Describe how the pattern grows from one figure to the next.

6. a. Copy and complete a T-table and an ordered pair list like the following in your notebook.

Figure Number	Number of Triangles	Ordered Pair
1	2	(1, 2)
2	4	(2, 4)
3	6	(3, 6)
4		
5		
6		

b. Graph the number of triangles for the first six figures. Use a sheet of graph paper from the Appendix.

c. Use the pattern of dots on your graph to predict the number of triangles for Figures 7, 8, and 9. Explain.

d. What is the rule that relates the number of triangles to the figure number?

e. Use your rule from question 6.d. to verify your predictions in question 6.c. Show your work.

7. a. Copy and complete a T-table like the following in your notebook.

Figure Number	Number of Squares	Ordered Pair
1	3	(1, 3)
2	4	(2, 4)
3	5	(3, 5)
4		
5		
6		

b. Graph the number of squares for the first six figures. Use a sheet of graph paper from the Appendix.

c. Extend your graph and predict the number of squares for Figures 7, 8, and 9.

d. What is the rule that relates the number of squares to the figure number?

e. Use your rule from question 7.d. to verify your answers in question 7.c. Show your work.

8. a. Copy and complete a T-table like the following.

Figure Number	Total Number of Pieces	Ordered Pair
1	5	(1, 5)
2	8	(2, 8)
3	11	(3, 11)
4		
5		
6		

b. Graph the number of pieces for the first six figures. Use a sheet of graph paper from the Appendix.

c. Extend your graph and predict the total number of pieces for Figures 7, 8, and 9.

d. What is the rule that relates the total number of pieces to the figure number?

e. Use your rule from question 8.d. to verify your answers in question 8.c. Show your work.

Check your answers on pages 106 to 108 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Turn to page 4 of the Practice and Homework Book and complete all the questions on the page.

Discuss your answers with your home instructor.

Challenge Activity

Business boomed when a heat wave hit after Joshua and Grace opened a lemonade stand on July 6. During the first week, including the weekend, they sold 3 more litres of lemonade each day than the day before. At the end of the week they had sold a total of 147 L of lemonade. How many litres of lemonade did they sell on July 12?

Check your answer on page 109 in the Appendix.

Conclusion

In this lesson you saw how objects, pictures, tables, and graphs can be used to represent patterns. You described relationships you saw in various patterns and used tables, rules, and graphs to extend these patterns.

When you were a small child, you probably did not realize that when you built towers or pyramids using blocks, you were really exploring mathematics! Did you know that the total number of blocks in the triangular tower shown is related to the number of layers?

Turn to Assignment Booklet 3A and complete the Lesson 2 Assignment.

When you are done, submit Assignment Booklet 3A to your teacher to be marked.

Lesson 3

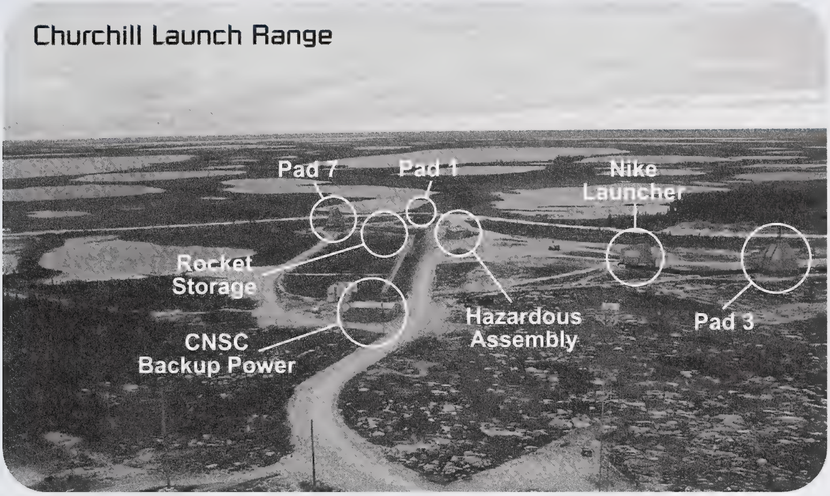
Beginning Algebra

Did you know that the word *algebra* was first used in mathematics as part of the title of a book about solving equations? The book was written by Arabic mathematician al-Khwarizmi. Al-Khwarizmi also wrote a book about arithmetic in which he presented Hindu-Arabic numerals (the standard 0 to 9 numerals you use today). This book helped persuade European mathematicians to use these numerals instead of Roman numerals. Hindu-Arabic numerals greatly simplified basic calculations.

In this lesson you will be introduced to algebraic expressions and equations. You will form word rules and algebraic expressions to describe how the quantities of objects in patterns are related to each other. You will see how graphs can be used to represent these relationships and how problems can be solved by finding the value of expressions or by reading the graphs. You will learn how to model and solve problems with equations. You will solve equations using blocks, pictures, and counting and then by using opposite operations.

Activity 1

Today you will be introduced to equations and their solutions.

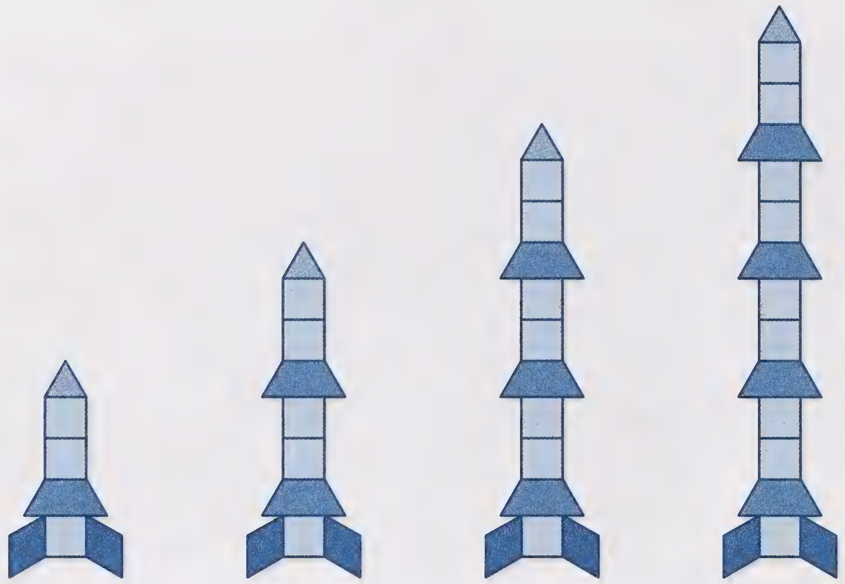


MANITOBA INDUSTRY, TRADE AND MINES/JOHN COKER

The Churchill Launch Range played an important part in Canada's proud history in aerospace research.

When Shytoo visited her cousin in Churchill, Manitoba, she learned that it was the site of the first rocket launch to mark the International Geophysical Year in 1957. This endeavour was so successful that NASA and Canada's National Research Council jointly funded and built a permanent rocket range. The range was ideally located for launching rockets to study Earth's space environment. Churchill Launch Range operated as a key part of the community for 25 years. The final rocket was fired on May 8, 1985.

On the plane back home to Iqaluit, Shytoo built the following models of rockets using pattern blocks. Use these pictures of her models to answer question 1.



1-stage rocket   2-stage rocket   3-stage rocket   4-stage rocket

1. Copy the following table into your notebook. Complete the first four rows for the first four rocket models.

Number of Stages	Number of Squares	Number of Trapezoids	Number of Rhombuses	Number of Triangles	Total Number of Blocks
1					
2					
3					
4					
5					
6					
7					

Use the table you completed in question 1 to answer questions 2 and 3.

2. Use words to describe the pattern that tell how the numbers in each column grow from model to model.

   a. number of stages
   b. number of squares
   c. number of trapezoids
   d. number of rhombuses
   e. number of triangles
   f. total number of blocks

3. The following rule can be written to describe how the number of squares relates to the number of stages.

$$(2 \times \text{number of stages}) + 1 = \text{number of squares}$$

   a. Write a rule that relates the number of trapezoids to the number of stages.

   b. Is the number of rhombuses related to the number of stages? Explain.

   c. Is the number of triangles related to the number of stages? Explain.

   d. Write a rule that relates the total number of blocks to the number of stages.

Check your answers on pages 109 and 110 in the Appendix.

4. Find how many of each shape of block you would need to build a model for a 7-stage rocket by using all three of the following methods.

   a. Complete the remaining rows in the table for question 1 one column at a time by using your patterns from question 2.

   b. Use the rules from question 3. Show how you would do this.

   c. Build the model with blocks or draw a picture of what it would look like. Then count the number of each shape of block that you used.

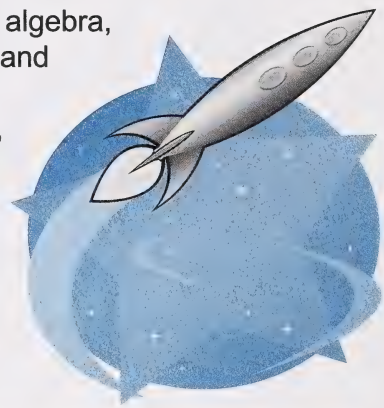
5. How many of each shape of block would you need to build a model for a 50-stage rocket? You may not want to draw a picture of it and count to find the number of each shape it would take. (Just imagine how big a model for a 50-stage rocket would be!) Instead, you might choose to use the method from either question 4.a. or question 4.b.

   a. Why would the method you used in question 4.b. be faster? Explain.

   b. Use the method from question 4.b. to find how many of each shape you would need to build a model for a 50-stage rocket.

Check your answers on pages 110 and 111 in the Appendix.

You have just learned the power of algebra! In algebra, the number of stages, the number of squares, and the number of trapezoids are called **variables** because the number of each of them changes, depending on the number of stages. A variable is represented by a letter of the alphabet—usually a letter that reminds you of what it stands for. The number of triangles and rhombuses are called **constants** because the number of them stays the same regardless of the number of stages.



An **algebraic expression** uses variables and constants along with other numbers and mathematical symbols to represent particular quantities (amounts of things).

To make an expression that describes a particular quantity, replace the key words and phrases with appropriate mathematical symbols.

Example

Write an algebraic expression that represents the rule from question 3.

Rule: The number of squares is always 1 more than twice the number of stages.

Step 1: Identify the key words and phrases.

The number of squares is **one more than twice** the **number of stages**. The word *twice* means "multiply by 2."

The number of squares is (one more than) (the number of stages) times 2

The number of squares is $1+$ (the number of stages) $\times 2$

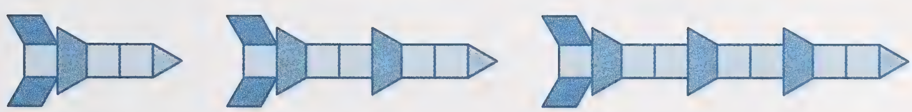
Step 2: You can represent the number of stages with the variable *n*. It is important to tell the meaning of all variables that you use. Stating what each variable stands for is called **defining the variables**. Now, the expression that relates the number of squares to the number of stages is $1+n\times 2$.

There are different ways to write the expression $1+n\times 2$.

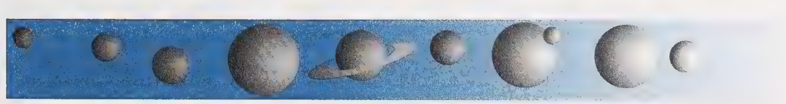
$1+(n\times 2)$ **or** $(n\times 2)+1$ **or** $(2\times n)+1$ **or** $2n+1$

All four expressions mean the same thing. Each tells you that to find the number of squares you need to make any particular rocket, double the number of stages you want and then add 1.

You can see the relationship $1+(n\times 2)$ in the models. For each model, there is 1 square in the base and 2 more squares for each stage.



$1+\ 1\times 2$ $\quad$ $1+\ 2\times 2$ $\quad$ $1+\ 3\times 2$



6. Write an expression that relates the number of each of the following pattern blocks to the number of stages.

   a. the number of rhombuses
   b. the total number of blocks

7. Write expressions to represent the patterns you found in question 2 for squares, rhombuses, and blocks. Use these expressions to answer each question.

   a. the number of squares in a 16-stage rocket
   b. the number of rhombuses in a 35-stage rocket
   c. the total number of blocks in a 21-stage rocket

Check your answers on page 112 in the Appendix.

An **equation** is an algebraic rule that relates two expressions that have the same value.

Equations can be used to solve problems. You can use square tiles to help you understand equations.

Example

Shytoo used 31 square pattern blocks to build one of her rocket models. How many stages did her rocket model have?

From your previous work, you know that the number of squares can be represented by the expression $1+(n\times 2)$. You found the value of this expression to get the number of squares that are needed to build a rocket of a given number of stages. However, this example asks for the opposite thing—you want to find how many stages the rocket has, given the total number of blocks used.

One method you might use is **guess and test**. Try different values of *n* in the expression until you discover the one that has a value of 31.

You will see that using algebra will take the guesswork out of solving the problem!

Make an equation by defining one more variable. Represent the number of squares with the variable *s*.

(the number of squares) **is** $1+(n\times 2)$

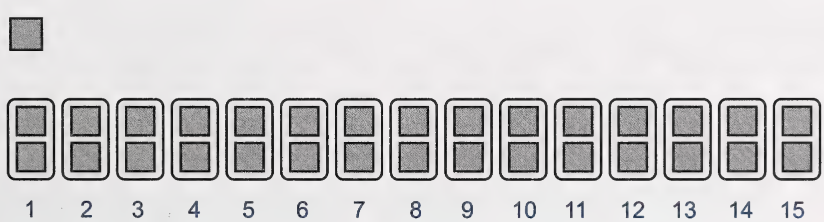
$$s = 1+(n\times 2)$$

Since you know the model has 31 squares, you can replace the variable *s* with the number 31 in the equation and write $31=1+(n\times 2)$.

You can solve the equation two ways:

Method 1: Using 31 Square Pattern Blocks

Take 31 square pattern blocks and arrange them so that there are many pairs of squares and one square all by itself. How many pairs of squares are in your arrangement of pattern blocks?



$n=15$, so the rocket model had 15 stages.

Method 2: Counting

Count 1 square for the base, and then skip count 2 more for each stage. Keep track of how many times you need to skip count by 2s to get to 31.

1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31
	+2	+2	+2	+2	+2	+2	+2	+2	+2	+2	+2	+2	+2	+2	+2
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

You can skip count 15 times, so the rocket had 15 stages.

You can verify your solution if $n = 15$.

$$s = 1 + (n \times 2)$$
$$s = 1 + (15 \times 2)$$
$$s = 1 + 30$$
$$s = 31$$

8. Use an equation to represent each of the following. Show how you can tell how many stages that particular rocket model must have and verify your answer.

   a. A rocket model is made using 4 rhombuses.
   b. A rocket model is made using 27 squares.
   c. A rocket model is made using 22 pattern blocks in total.

Check your answers on pages 112 to 114 in the Appendix.

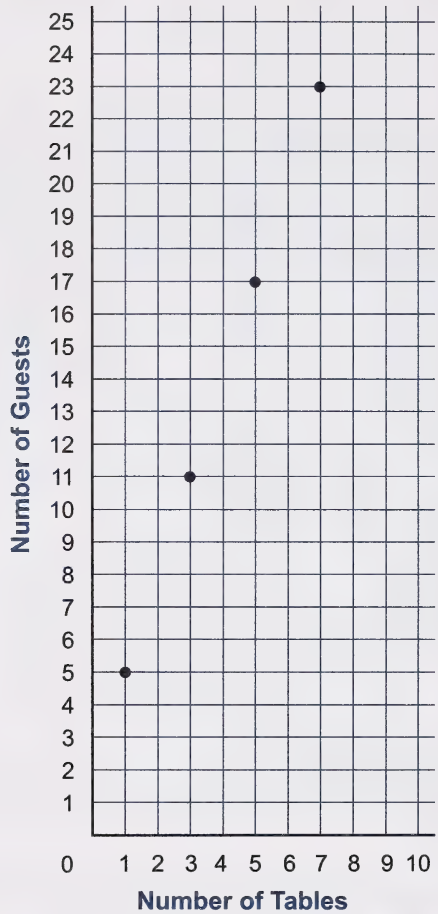
Activity 2

Today you will explore equations and their graphs.

Tabletown rents tables in the shapes of regular polygons. All the tables of any particular shape are the same size and can be joined along one edge to form longer tables.

JoAnn is helping her family plan a reunion. They have rented some tables of one of the shapes from Tabletown.

JoAnn plotted the number of guests that can be seated at 1, 3, 5, and 7 tables. Her graph is shown.



1. Show how JoAnn can use her graph to find the number of guests that can be seated at 2, 4, and 6 tables.

2. a. Use the information from the graph to copy and complete a table like the following in your notebook.

Number of Tables (t)	Number of Guests (g)
1	
2	
3	
4	
5	
6	
7	

b. How many more guests can be seated each time another table is added?

c. Extend the table three more rows, and use the pattern you found in question 2.b. to show the number of guests that can be seated at 8, 9, and 10 tables.

Check your answers on pages 114 and 115 in the Appendix.

3. a. Write a rule to show how the number of guests that can be seated is related to the number of tables.

b. Write an equation you can use to find the number of guests (g) that can be seated at tables (t).

4. a. Verify your answers to question 2.c. by using your equation from question 3.b.

b. Extend the graph and read the values for 8, 9, and 10 tables.

5. a. Use the table from question 2.a. to determine the shape of the tables.

b. Draw a diagram that shows how 7 of the tables can be arranged to form one long table to seat the number of guests you found in question 2.a.

6. The equation $g = 2t + 2$ shows how the number of guests (g) that can be seated is related to the number of tables (t) used for a different shape of table available from Tabletown.

a. Copy and complete the following table in your notebook to show the number of guests that can be seated at 1, 2, 3, 4, or 5 tables.

Number of Tables (t)	**Number of Guests** $g = 2t + 2$
1	
2	
3	
4	
5	

b. How many additional guests can be seated for each table that is added?

c. Label and complete a graph to show the number of guests that can be seated at 1, 2, 3, 4, or 5 tables. Use a sheet of graph paper from the Appendix.

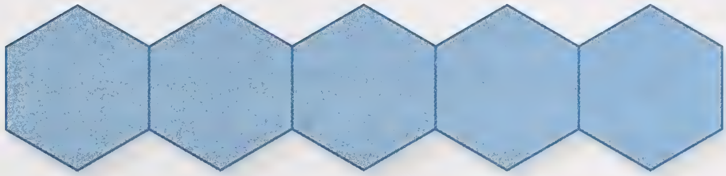
d. Use the equation to find the number of guests that can be seated at 10 tables.

e. Verify your answer to question 6.d. by extending the graph you made in question 6.c.

f. Draw a picture to show how the 10 tables would look when arranged to seat the number of guests you found in question 6.d.

Check your answers on pages 115 to 119 in the Appendix.

7. The following picture shows how a different shape of table available from Tabletown can be arranged to form a longer table.



a. Complete a table to show how the number of guests that can be seated is related to the number of tables.

b. Write an equation to show how the number of guests that can be seated is related to the number of tables used. (Remember to define the variables.)

8. Tables shaped like equilateral triangles are also available from Tabletown.

a. Draw a picture to show how 6 of these tables would look when arranged to form a longer table, and show where the guests would be seated.

b. Write an equation to show how the number of guests (g) that can be seated is related to the number of tables used (t).

9. In this activity you worked with four different shapes of tables. For each shape, you formed an equation that related the number of tables used to the number of guests that can be seated around them. Explain how an equation can be formed for a table of any given number of sides. (**Hint:** Compare the equations for the four different shapes of tables. Look for a pattern in how they are alike.)

Check your answers on page 119 in the Appendix.

Activity 3

Today you will investigate strategies for solving equations. These strategies will involve balancing equations.

In the first two activities in this lesson you used equations to represent relationships seen in patterns modelled with objects, pictures, and graphs. You saw the power of using these equations to solve problems by predicting and verifying unknown values in the pattern. One way to think about equations is to visualize them as balancing acts.

Suppose you had a balance scale and some blocks, each of equal mass, as shown in the following picture.

Balance Scale

Blocks

1. **a.** Put 5 blocks on each pan. Will the scale be balanced? Draw a picture to show your work. Explain.

   b. Add 6 blocks to each pan. Show the results. Will the scale stay balanced? Explain.

   c. Remove 7 blocks from each pan. Show the results. Will the scale stay balanced? Explain.

   d. Put 3 times as many blocks on each pan as there are now. Show the results. Will the scale stay balanced? Explain.

   e. Remove half of the blocks from each pan. Show the results. Will the scale stay balanced? Explain.

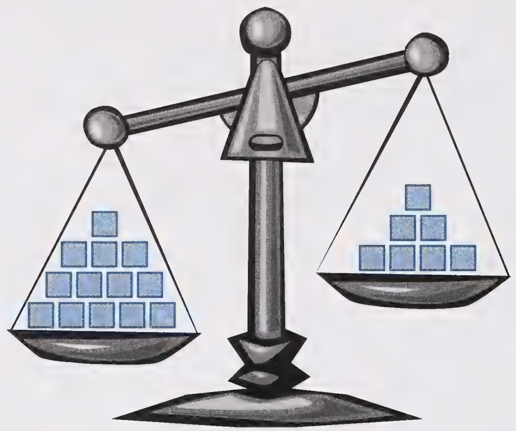
Check your answers on pages 120 and 121 in the Appendix.

Question 1 shows you that as long as you use the same operation on both pans, the scale stays balanced. When solving equations, you must keep a similar kind of balance between the left side and the right side.

Example

Carlos had 13 books on his shelf after he added more books to the 7 books that were already on his shelf. To find how many books Carlos added to his shelf, use the equation $13 = 7 + \boxed{}$.

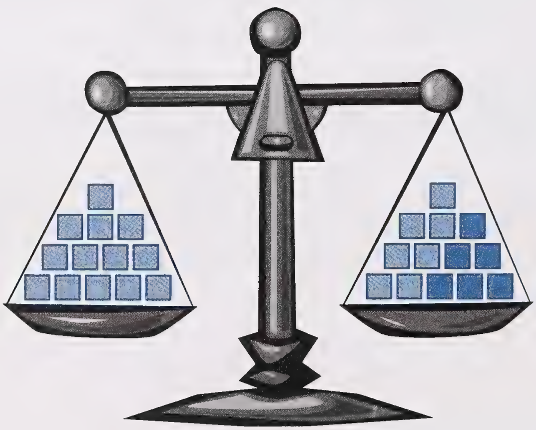
To solve this equation, think of a balance scale with 13 blocks in the left pan and 7 blocks in the right pan. How many blocks must you add to the right pan to balance the scale?



You could add blocks one at a time and count as you add each block. You would need to add 6 blocks.

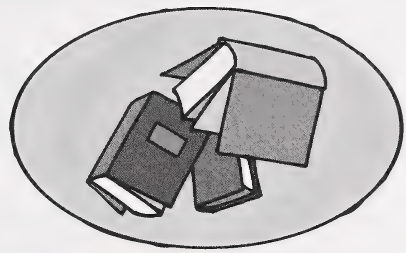
$$13 = 7 + \boxed{6}$$

This means that Carlos added 6 books to his shelf.



Equations can also be solved by using a "working-backwards" strategy that uses the **inverse relationship** between addition and subtraction or between multiplication and division. You can use this working-backwards strategy to solve the equation $13 = 7 + \square$.

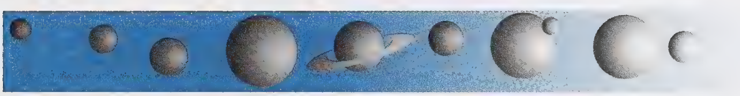
To find the number you add to 7 to get 13, you can subtract 7 from 13. (Here's where it's helpful to know your basic facts!)



$13 - 7 = 6$

Therefore, $13 = 7 + \boxed{6}$.

This means that Carlos added 6 books to his shelf.



2. Solve each of the following equations in two ways: by drawing pictures that show how you can use a balance scale and blocks and by using a working-backwards strategy.

a. $8 + \square = 15$

b. $\square \times 3 = 18$

c. $12 = 20 - n$

d. $7 = 21 \div t$

Check your answers on pages 121 to 123 in the Appendix.

You can solve problems that involve pre-algebra strategies by using a working-backwards strategy. You can also check your answer by working forwards.

Example

I am a number. Multiply me by 3. Subtract 10 from the product. You end with 14. What number am I?

Solution (Working Backwards)

Begin with14. Add 10.

$$14 + 10 = 24$$

Divide by 3.

$$24 \div 3 = 8$$

The number is 8.

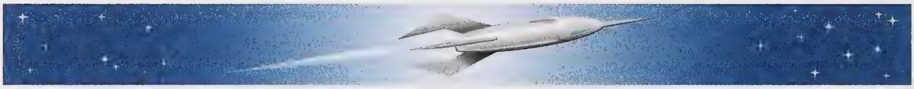
Check (Working Forwards)

Begin with 8. Multiply by 3.

$$8 \times 3 = 24$$

Subtract 10.

$$24 - 10 = 14$$



3. Turn to page 86 in your textbook to Solving Problems Using Pre-Algebra Strategies. Do questions 1 to 4.

  a. Solve each problem by using a working-backwards strategy.

  b. Check your answers by working forwards. This is the normal way you work through the problem.

4. Do question 5 on page 86 in your textbook.

  a. Try several beginning numbers and work forwards until you notice a pattern.

  b. What final answer will you always get? Explain.

Check your answers on pages 123 to 126 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Turn to page 40 of the Practice and Homework Book and complete questions 1 to 5.

Discuss your answers with your home instructor.

Challenge Activity

The town recreation department needed to buy some new equipment for the summer sports program. The sporting goods store sells footballs for \$25.00, soccer balls for \$20.00, tennis balls for \$2.50, and baseballs for \$5.00. When the sports director got to the store, she realized that she had forgotten her list, but she remembered that she needed to buy the following amounts of balls:

- twice as many tennis balls as baseballs
- 3 times as many baseballs as soccer balls
- half as many soccer balls as footballs
- 12 footballs

How many balls did the director buy? What is the total cost for them?

Check your answers on page 126 in the Appendix.

Conclusion

In this lesson you were introduced to algebraic expressions and equations. You formed word rules and algebraic expressions to describe how the quantities of objects in patterns are related to each other. You used graphs to represent these relationships. You solved problems by finding values of expressions and by reading graphs. You learned how to model and solve problems with equations. You solved equations using blocks, pictures, and counting and then by using opposite operations.

Some of the procedures for balancing equations and using inverse operations to find solutions were presented over 1200 years ago by al-Khwarizmi in his book titled *Al-Kitah Al-jabr wa'l muqabalah*. Can you find the word we know as *algebra* in the title?

Turn to Assignment Booklet 3B and complete
the Lesson 3 Assignment.

Keep Assignment Booklet 3B until you have completed the entire booklet.

Module Summary

In Module 3 you reinforced your understanding of patterns for solving problems. You extended your understanding of factors and multiples. You explored an ancient method for finding prime numbers and you learned about prime factorization.

You were introduced to algebraic expressions and saw how they can be used to represent patterns using objects, pictures, and graphs. You modelled problems with equations and then solved them by using blocks, pictures, and counting.

In this module you explored mathematics essentially the same way as mathematicians do, by searching for and describing patterns. The search for patterns was a quest throughout history. Early peoples looked into the night sky and saw patterns in the stars around which they wove myths and stories. Modern astronomers and cosmologists look into the night sky and use sophisticated mathematics to help unravel the origins of the universe.

Turn to Assignment Booklet 3B and complete the Numbers in the News project.

When you are done, submit Assignment Booklet 3B to your teacher to be marked.

Keystrokes

Take out your calculator and complete the following exercises. They will help you review some of the ideas you have learned in Module 3.

Funky Feature: Multiple Mystery

1. a.
   - Enter 247 in your calculator.
   - Multiply by 7.
   - Multiply that answer by 11.
   - Multiply that answer by 13.

   Copy and complete the following table to show your work.

Keystrokes	ON/C	247	×	7	×	11	×	13	=
Display	0	247	247		1729	11	19019		247247

   b. Describe your results. (Pretty cool, don't you think?)

   c. Try the same operations using a few more three-digit numbers. Show the results in similar tables.

   d. Explain why this works as it does. (**Hint:** Use what you have learned about multiples. Think about what single number you could multiply your three-digit number by to get the same results.)

2. a. Reverse the problem. Use your answer to question 1.d. to predict what will happen if you enter a six-digit number that has the same pattern as your resulting numbers in question 1 and then divide by 7. Divide that answer by 11. Divide that answer by 13.

b. Copy and complete the following table to check your prediction with your own six-digit number.

Keystrokes	ON/C		÷	7	÷	11	÷	13	=
Display	0			7		11			

Check your answers on page 127 in the Appendix.

Funky Feature: Happy Birthday!

3. Complete the following table using your date of birth.

	Keystrokes	**Display**
Enter the number of the month you were born in. (Jan. = 1 Feb. = 2, …, Dec. = 12)		
Multiply by 5.	×	
	5	
Add 5.	+	
	5	
Multiply by 20.	×	
	20	
Add the day of the month on which you were born. (1st = 1, 2nd = 2, …, 31st = 31)	+	
Subtract 100.	−	
	100	
	=	

If the final number has three digits, the first digit gives the month you were born in, and the second and third digits give the day of the month you were born on.

If the final number has four digits, the first and second digits give the month you were born in, and the third and fourth digits give the day of the month you were born on.

4. Complete a table like the following by doing the inverse operations. The number you end up with will show the month you were born in.

	Keystrokes	Display
Enter the final number on the display in question 1.		

Try this with the birth dates of family members and friends.

Check your answers on page 128 in the Appendix.

Review

The activities in this lesson will help you review and apply what you learned in Module 3 and prepare for the final test. Discuss with your home instructor when you should begin the Review and how much of the Review you should complete.

1. Turn to page 67 of your textbook. Do questions 1 and 2 of On Your Own. Show your work.

2. Do questions 1 to 8 of Practise Your Skills on page 67 of the textbook.

If you need help with questions 1 and 2, look back at Lesson 1, where you learned about finding multiples and the greatest common multiple. If you feel you need more practice, do questions 3 and 4.

3. Turn to page 96 of your textbook to Skill Bank from This Unit. Do question 1.

4. Turn to page 123 of your textbook to Skill Bank Looking Back. Do question 6.

Check your answers on pages 129 to 132 in the Appendix.

5. Turn to page 68 of your textbook to Exploring Factors. Do the problem at the top of the page. Show your work.

6. Turn to page 70 of your textbook to Practise Your Skills. Do question 1.

If you need help with questions 5 and 6, look back at Lesson 1, where you learned about finding factors and the greatest common factor. If you feel you need more practice, do questions 7, 8, and 9.

7. Turn to page 96 of your textbook to Skill Bank from This Unit. Do questions 2 and 3.

8. Turn to page 123 of your textbook to Skill Bank Looking Back. Do question 7.

9. Turn to page 145 of your textbook to Skill Bank Looking Back. Do question 1.

Check your answers on pages 132 to 135 in the Appendix.

10. Turn to page 70 of your textbook to Practise Your Skills. Do question 2. If the number is not prime, give at least one pair of factors that prove it is composite.

If you need help with question 10, look back at Lesson 1, where you learned about prime and composite numbers. If you feel you need more practice, do questions 11 and 12.

11. Turn to page 96 of your textbook to Skill Bank from This Unit. Do question 4.

12. Turn to page 145 of your textbook to Skill Bank Looking Back. Do question 2.

Check your answers on page 136 in the Appendix.

13. Turn to pages 69 and 70 of your textbook to On Your Own. Do question 1 using the factor-tree method. Do question 2 using the repeated division by primes to find the prime factorization.

If you need help with question 13, look back at Lesson 1, where you learned about prime factorization. If you feel you need more practice, do question 14.

14. Turn to page 145 of your textbook to Skill Bank Looking Back. Do question 3 by making a factor tree and by using the repeated division by primes for each number.

Check your answers on pages 136 to 138 in the Appendix.

15. Turn to page 9 of your textbook. Look at the tables that show sunrise and sunset times.

a. What is the rule for the pattern in the T-table for sunrise times?

b. What is the rule for the pattern in the T-table for sunset times?

c. Use tables like the following to continue each pattern for four more entries.

Date	Sunrise Time (A.M.)
Sept 17	
Sept 18	
Sept 19	
Sept 20	

Date	Sunset Time (P.M.)
Sept 19	
Sept 20	
Sept 21	
Sept 22	

16. Turn to page 26 of your textbook to Finding Rules. Do questions 1 to 4.

17. Turn to page 27 of your textbook to On Your Own. Do questions 1 to 4 and 6 to 8.

If you need help with questions 15 to 17, look back at Lesson 2, where you learned about making T-tables and finding patterns. If you feel you need more practice, do questions 18 and 19.

18. Turn to page 15 of your textbook to Practise Your Skills. Do questions 1 and 2.

19. Turn to page 21 of your textbook to Practise Your Skills. Do questions 1 to 3.

Check your answers on pages 138 to 140 in the Appendix.

Turn to page 14 of your textbook to Flying Kites. Use Figure 1, Figure 2, Figure 3, and the T-tables to answer questions 20 to 22.

20. a. Copy and complete the following T-table in your notebook.

Figure Number	Number of Triangles
1	4
2	6
3	8
4	
5	
6	

b. Graph the number of triangles.

c. What rule relates the figure number to the number of triangles?

21. a. Copy and complete the following T-table in your notebook.

Figure Number	Number of Rhombuses
1	1
2	2
3	3
4	
5	
6	

b. Graph the number of rhombuses.

c. What is the rule that relates the figure number to the number of rhombuses?

22. a. Copy and complete the following T-table in your notebook.

Figure Number	Total Number of Pieces
1	5
2	8
3	11
4	
5	
6	

b. Graph the total number of pieces for each figure.

c. What is the rule that relates the figure number to the total number of pieces?

If you need help with questions 20 to 22, look back at Lesson 2, where you learned about finding patterns, extending T-tables, and making graphs. If you feel you need more practice, do question 23.

23. Turn to page 18 of your textbook. Do On Your Own questions 1 and 2.

Check your answers on pages 140 to 142 in the Appendix.

24. Turn to page 89 of your textbook to On Your Own. Do questions 1 to 4.

25. Turn to page 88 of your textbook to Solving Equations.

a. Do questions 1, 3, 5, 6, and 9 by drawing pictures that show how you can use a balance scale and blocks to solve each equation.

b. Do questions 2, 4, 7, 8, and 10 by using a working-backwards strategy.

26. Turn to page 87 of your textbook to On Your Own. Do question 1.

If you need help with questions 24 to 26, look back at Lesson 3, where you learned about solving equations. If you feel you need more practice, do questions 27 and 28.

27. Turn to page 89 of your textbook to Practise Your Skills.

  a. Do questions 1, 3, and 4 by drawing pictures that show how you can use a balance scale and blocks to solve each equation.

  b. Do questions 2, 5, 6, 7, and 8 by using a working-backwards strategy.

28. Turn to page 87 of your textbook to Practise Your Skills. Complete the table. Show your work.

29. Turn to pages 30 and 31 of your textbook to Problem Bank. Do questions 1 to 5.

Check your answers on pages 142 to 155 in the Appendix.

If you still feel you need more practice for Module 3, you may do the following pages in the Practice and Homework Book.

- pages 22 to 25 (review of Lesson 1)
- pages 2 to 11 (review of Lesson 2)
- pages 38 to 41 (review of Lesson 3)

If you need additional work to master the material in this module, work through the following lessons on the Mathematics 6 Companion CD:

- Lesson 7: Factors, Multiples, and Prime Factorization
- Lesson 8: Summarizing and Extending Patterns
- Lesson 11: Balancing Equations

After you complete each lesson, you can print out an activity by clicking on the Activity button at the bottom of the screen.

Ask you home instructor to print out the solutions to the questions in each activity by clicking on the Parent Notes button at the bottom of the screen. Discuss your answers with your home instructor.

Just the Facts

Ask your home instructor to time you as you complete the following timed drill. Your goal is to complete all 25 questions in two minutes. At the end of two minutes, count how many questions you were able to complete. Then use the Answer Key in the Appendix to mark the drill, and record your score in the space provided. Before you move on, go back and complete any questions you did not finish.

Multiplication and Division Facts

$0 \times 2 =$	$12 \div 6 =$	$\begin{array}{r} 3 \\ \times 5 \\ \hline \end{array}$	$9\overline{)27}$	$7 \times 0 =$
$35 \div 5 =$	$\begin{array}{r} 8 \\ \times 1 \\ \hline \end{array}$	$3\overline{)0}$	$6 \times 5 =$	$35 \div 7 =$
$\begin{array}{r} 1 \\ \times 2 \\ \hline \end{array}$	$3\overline{)12}$	$9 \times 7 =$	$27 \div 3 =$	$\begin{array}{r} 3 \\ \times 8 \\ \hline \end{array}$
$7\overline{)56}$	$2 \times 9 =$	$30 \div 5 =$	$\begin{array}{r} 0 \\ \times 3 \\ \hline \end{array}$	$2\overline{)2}$
$4 \times 7 =$	$8 \div 4 =$	$\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$	$1\overline{)2}$	$1 \times 4 =$

Multiplication and Division Facts

Number completed in 2 minutes: ______
Number correct in 2 minutes: ______

Record your score on the Just the Facts Progress Chart.

Mental Math

Kylee often makes long-distance phone calls to her grandma, who lives in Slave Lake. She is allowed 100 minutes per month without extra charge, so she keeps track of the time she spends on each call. She made the following calls in November.

Date	Minutes
Nov. 2	5
Nov. 7	17
Nov. 11	8
Nov. 14	25
Nov. 18	3
Nov. 20	2

After the phone call on November 20, Kylee wanted to find the total number of minutes for her November calls. There are different ways to find the total.

Method 1: Add the numbers in the order they are listed.

$$\begin{array}{rr}
5 & \\
17 & 5+17=22 \\
8 & +8=30 \\
25 & +25=55 \\
3 & +3=58 \\
\underline{2} & +2=60 \\
60 &
\end{array}$$

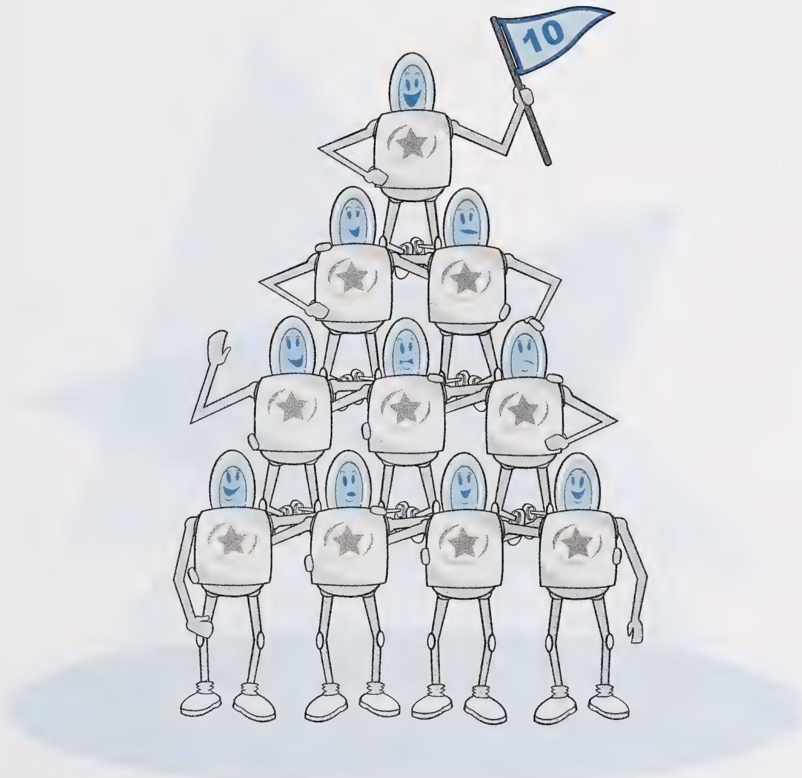
Method 2: Find pairs of numbers in the list with sums that are multiples of 10. Then add the multiples of 10.

$$\begin{array}{r} 5 \\ 17 \\ 8 \\ 25 \\ 3 \\ \underline{2} \\ 60 \end{array} \qquad 5 + 25 = 30 \qquad 17 + 3 = 20 \qquad 8 + 2 = 10$$

Add the multiples of 10.

30 + 20 = 50; 50 + 10 = 60

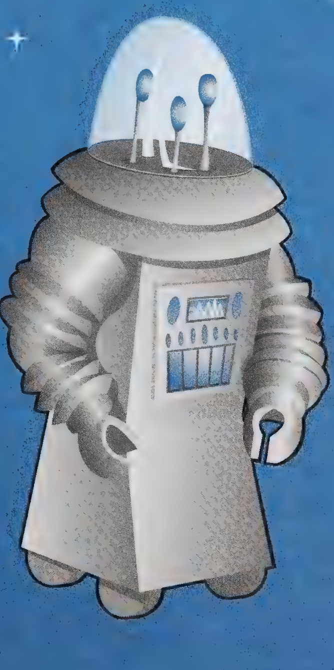
The second method uses sums that are multiples of 10 and the associative property. Sums that are multiples of ten use the addition basic facts for 10. The **associative property** is the rule that allows you to change the order in which you add a list of numbers. Remember, you can use this strategy to mentally add numbers faster, even if all the numbers in a list do not form pairs that make multiples of 10.



Mathematics 6

Appendix

Glossary
Answer Key
Image Credits
Learning Aids



Glossary

algebraic expression: an expression that uses variables and constants to represent particular quantities

common factor: a factor that divides into two or more given numbers

common multiple: a number that is a multiple of two or more whole numbers

composite number: a whole number with more than two factors

constant: a number in an algebraic expression that represents a quantity that does not change in value

equation: a statement of the equality of two quantities

factor: one whole number that divides into another whole number

Fundamental Theorem of Arithmetic: Any whole number can be written as a product of a unique set of prime numbers.

guess and test: a problem-solving strategy that uses trials to test possible answers

greatest common factor: the largest in the list of factors that divide into two or more given numbers

inverse operation: an operation that undoes a given operation

least common multiple: the smallest common multiple

multiple: a number formed by multiplying a given whole number by another whole number

ordered pair: a pair of numbers that represent a point on a graph

prime factorization: the product of prime numbers to which a given whole number is equal

prime number: a whole number with exactly two factors

relatively prime: two whole numbers that have only 1 as a common factor

rhombus: a parallelogram with four equal sides

trapezoid: a quadrilateral with one pair of parallel sides

unique: one of a kind

variable: a letter in an algebraic expression that indicates a value that can change

Answer Key

Lesson 1: Whole Number Patterns

Activity 1

1. You should have drawn circles around 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, and 96.

2. You should have drawn triangles around 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, and 96.

3. There are four common multiples of 6 and 8 on the hundred chart: 24, 48, 72, and 96. These are the first four multiples of 24.

4. The LCM of 6 and 8 is 24.

5. Every fourth multiple of 6 is a multiple of 24 and every third multiple of 8 is a multiple of 24. All multiples of 24 are multiples of both 6 and 8.

6. The two teams completed a menu cycle together every 24 days.

7. If the mission began on September 2, the two teams first completed a menu cycle together on September 25.

   Team A: Sept. 7, $\xrightarrow{+6}$ 13, $\xrightarrow{+6}$ 19, $\xrightarrow{+6}$ 25

   Team B: Sept. 9, $\xrightarrow{+8}$ 17, $\xrightarrow{+8}$ 25

8. The two teams completed a meal cycle together three times before December.

   Team A: Sept. 7, $\xrightarrow{+6}$ 13, $\xrightarrow{+6}$ 19, $\xrightarrow{+6}$ 25 $\xrightarrow{+6}$ Oct. 1, $\xrightarrow{+6}$ 7, $\xrightarrow{+6}$ 13, $\xrightarrow{+6}$ 19 $\xrightarrow{+6}$ 25, $\xrightarrow{+6}$ 31, $\xrightarrow{+6}$ Nov. 6, $\xrightarrow{+6}$ 12, $\xrightarrow{+6}$ 18, $\xrightarrow{+6}$ 24, $\xrightarrow{+6}$ 30

Team B: Sept. 9, $\xrightarrow{+8}$ 17, $\xrightarrow{+8}$ 25 $\xrightarrow{+8}$ Oct. 3, $\xrightarrow{+8}$ 11, $\xrightarrow{+8}$ 19 $\xrightarrow{+8}$ 27, $\xrightarrow{+8}$ Nov. 4, $\xrightarrow{+8}$ 12 $\xrightarrow{+8}$ 20, $\xrightarrow{+8}$ 28

9. a. The seven dates before December that Team C ended a meal cycle are listed below. The dates that all three teams completed a menu cycle together are circled.

Team A: Sept. 7, $\xrightarrow{+6}$ 13, $\xrightarrow{+6}$ 19, $\xrightarrow{+6}$ 25 $\xrightarrow{+6}$ Oct. 1, $\xrightarrow{+6}$ 7, $\xrightarrow{+6}$ 13 $\xrightarrow{+6}$ 19 $\xrightarrow{+6}$ 25 $\xrightarrow{+6}$ 31 $\xrightarrow{+6}$ Nov. 6, $\xrightarrow{+6}$ 12 $\xrightarrow{+6}$ 18, $\xrightarrow{+6}$ 24, $\xrightarrow{+6}$ 30

Team B: Sept. 9, $\xrightarrow{+8}$ 17, $\xrightarrow{+8}$ 25 $\xrightarrow{+8}$ Oct. 3, $\xrightarrow{+8}$ 11, $\xrightarrow{+8}$ 19 $\xrightarrow{+8}$ 27, $\xrightarrow{+8}$ Nov. 4, $\xrightarrow{+8}$ 12 $\xrightarrow{+8}$ 20, $\xrightarrow{+8}$ 28

Team C: Sept. 13, $\xrightarrow{+12}$ 25 $\xrightarrow{+12}$ Oct. 7, $\xrightarrow{+12}$ 19 $\xrightarrow{+12}$ 31, $\xrightarrow{+12}$ Nov. 12 $\xrightarrow{+12}$ 24

b. All three teams completed a menu cycle together every 24 days.

c. September 25 was the first day all three teams completed a menu cycle together.

d. The menu cycle of Team C (12 days long) overlapped so well with the others because 12 is a factor of 24 (the LCM of 6 and 8).

e. To show the LCM of 6, 8, and 12, cover the multiples of 6 with counters of one colour, cover the multiples of 8 with counters of a second colour, and cover the multiples of 12 with counters of a third colour. The first number covered by all three colours of counters would be 24.

Activity 2

1. **a.** Philip could have sorted his cookies into the following groups:

    - 24 groups of 1 or 1 group of 24

    - 2 groups of 12 or 12 groups of 2

    - 3 groups of 8 or 8 groups of 3

    - 4 groups of 6 or 6 groups of 4

   b. Helen could have sorted her cookies into the following groups:

    - 30 groups of 1 or 1 group of 30

    - 2 groups of 15 or 15 groups of 2

    - 3 groups of 10 or 10 groups of 3

- 5 groups of 6 or 6 groups of 5

2. Both Philip and Helen can wrap their cookies individually, or they can put either 2, 3, or 6 cookies in each bag.

3. The largest group of cookies that can be made with both 24 and 30 cookies is by putting 6 cookies in each group.

4. Factors of 24: 1, 2, 3, 4, 6 8, 12, 24

Factors of 30: 1, 2, 3, 5, 6 10, 15, 30

5. Find and list all the factors of one number and then all the factors of the other number. Test for possible factors of each number by trying to divide by 2, then by 3, then by 4, and so on. If a number divides evenly, both the divisor and the quotient are factors. (Factors come in pairs whose product is the starting number.) Stop testing when the factors begin to repeat.

Look for factors that appear in both lists. Select the largest factor from the list. The largest factor is the GCF.

6. a. Factors of 20: 1, 2, 4, 5, 10, 20
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Common factors of 20 and 36: 1, 2, 4

$20 \div 2 = 10$	$36 \div 2 = 18$
$20 \div 3$ (Doesn't divide evenly.)	$36 \div 3 = 12$
$20 \div 4 = 5$	$36 \div 4 = 9$
$20 \div 5 = 4$ (Factors start to repeat.)	$36 \div 5$ (Doesn't divide evenly.)
	$36 \div 6 = 6$ (Factors start to repeat.)

b. The GCF of 20 and 36 is 4.

7. Such a pair of numbers doesn't exist. All pairs of numbers must have at least a single common factor, the number 1. This is because all numbers are divisible by 1.

8. a. Philip, Helen, and Neil can all make packages with either 1, 2, 3, or 6 cookies in each. To answer this question, find all of the common factors of 18, 24, and 30.

- Factors of 18: 1, 2, 3, 6, 9, 18
- Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
- Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
- Common factors of 18, 24, and 30: 1, 2, 3, 6

b. The largest packages that they all can make will have 6 cookies in each package. (6 is the GCF of 18, 24, and 30.)

Activity 3

1. The number 1 is not a prime number because it has only one factor. Prime numbers have two factors.

2. a. The number 2 is a prime number because it has only two factors, itself and 1.

b. Numbers that are divisible by 2 are called even numbers.

c. The only number divisible by 2 that is prime is 2 itself. (So, 2 is the only even prime number!) If any other number has 2 as a factor, then it has at least three factors: itself, 1, and 2. Therefore, once a prime number is circled on the chart, all other multiples of it must be crossed out.

d. The number 1 and the multiples of 2 are crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15	~~16~~	17	~~18~~	19	~~20~~
21	~~22~~	23	~~24~~	25	~~26~~	27	~~28~~	29	~~30~~
31	~~32~~	33	~~34~~	35	~~36~~	37	~~38~~	39	~~40~~
41	~~42~~	43	~~44~~	45	~~46~~	47	~~48~~	49	~~50~~
51	~~52~~	53	~~54~~	55	~~56~~	57	~~58~~	59	~~60~~
61	~~62~~	63	~~64~~	65	~~66~~	67	~~68~~	69	~~70~~
71	~~72~~	73	~~74~~	75	~~76~~	77	~~78~~	79	~~80~~
81	~~82~~	83	~~84~~	85	~~86~~	87	~~88~~	89	~~90~~
91	~~92~~	93	~~94~~	95	~~96~~	97	~~98~~	99	~~100~~

A pattern is made by the multiples of 2. Every second column is made up completely of even numbers. Therefore, you can cross out those five complete columns (except for the number 2 itself).

3. a. The number 3 is a prime number because it has two factors, itself and 1.

b. The number 1, the multiples of 2, and the multiples of 3 are crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	25	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	35	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49	~~50~~
~~51~~	~~52~~	53	~~54~~	55	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	65	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	77	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	85	~~86~~	~~87~~	~~88~~	89	~~90~~
91	~~92~~	~~93~~	~~94~~	95	~~96~~	97	~~98~~	~~99~~	~~100~~

The multiples of 1 and 3 are not prime since each multiple has more than two factors. The multiples of 3 are all located diagonally (from lower left to upper right) to each other.

c. Some of the multiples of 3 were already crossed out on the chart because they are also multiples of 2.

4. a. The number 5 is a prime number because it has only two factors, itself and 1.

b. The number 1, the multiples of 2, the multiples of 3, and the multiples of 5 are crossed out.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	49	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	77	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
91	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

All of the numbers in the fifth and tenth columns are multiples of 5.

c. Some of the multiples of 5 were already crossed out on the chart because they are also multiples of 2 and/or multiples of 3.

5. a. The number 7 is a prime number because it has only two factors, itself and 1.

b. The number 1, the multiples of 2, the multiples of 3, the multiples of 5, and the multiples of 7 are crossed out on the following chart.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

From any multiple of 7, drop down one row and slide three columns left. (Or drop down two rows and slide one column right.)

c. Some of the multiples of 7 were already crossed out on the chart because they are also multiples of 2, multiples of 3, and/or multiples of 5.

6. Factors come in pairs. If you increase the first factor to 20, the other factor will decrease to 5, but the 20 by 5 rectangle is just a rotation of the 5 by 20 rectangle that is already drawn. Similarly, if you increase the first factor to 25, the other factor will decrease to 4, but the 4 by 25 rectangle is already drawn, and so on.

7. If either factor in a factor pair for 100 is greater than 10, then the other factor in the pair is less than 10.

8. You don't have to test prime factors greater than or equal to 11, because if one factor is 11, then the other factor in the pair is less than 11, and you have already tested the prime factors that are less than 11.

9. There are 25 prime numbers less than 100. They are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

Activity 4

1. Lyall's final score was 10.

   $1 \text{ red} \times 1 \text{ green} = 2 \times 5 = 10$

2. Aaron's final score was 90.

   $1 \text{ red} \times 2 \text{ white} \times 1 \text{ green} = 2 \times 3 \times 3 \times 5 = 90$

3. Chelsea's final score was 30.

   $1 \text{ red} \times 1 \text{ white} \times 1 \text{ green} = 2 \times 3 \times 5 = 30$

4. a. The greatest final score you could get if you ring three pylons is 125. You would have to ring 3 green pylons.

   $5 \times 5 \times 5 = 125$

   b. The least final score you could get if you ring three pylons is 8. You would have to ring three red pylons.

   $2 \times 2 \times 2 = 8$

5. a. The greatest final score you could get if you ring four pylons is 625. You would have to ring four green pylons.

$$5 \times 5 \times 5 \times 5 = 625$$

b. The least final score you could get if you ring four pylons is 16. You would have to ring four red pylons.

$$2 \times 2 \times 2 \times 2 = 16$$

6. a. Reg might begin by saying, "I know 20 is a multiple of 5 because it ends in 0." That means Benita won a green token. Now, $20 \div 5 = 4$, and $4 = 2 \times 2$. That means Benita also won two red tokens.

b. No. Benita could not have won tokens other than one green and two red. $2 \times 2 \times 5$ is the only combination of token values that gives a product of 20.

7. a.

Name	Final Score	Work Done to Find Values of Tokens	Values of Tokens (least to greatest)
Tyler	75	$75 \div 3 = 25$ and $25 = 5 \times 5$	$3 \times 5 \times 5$
Damon	24	$24 \div 2 = 12$, $12 = 3 \times 4$, and $4 = 2 \times 2$	$2 \times 2 \times 2 \times 3$
Vita	45	$45 \div 5 = 9$ and $9 = 3 \times 3$	$3 \times 3 \times 5$
Saul	54	$54 \div 2 = 27$, $27 = 3 \times 9$, and $9 = 3 \times 3$	$2 \times 3 \times 3 \times 3$
Cora	90	$90 \div 2 = 45$, $45 = 5 \times 9$, and $9 = 3 \times 3$	$2 \times 3 \times 3 \times 5$

b. No. A score made from ringing four pylons will not always be greater than a score made from ringing three pylons.

8. No, you are only able to find one set of tokens for any person in question 7.

9. a.

$$42$$
$$6 \times 7$$
$$2 \times 3 \times 7$$

So, $42 = 2 \times 3 \times 7$.

or

$2\overline{)42} = 2 \times 3 \times 7$

$3\overline{)21}$

$7\overline{)7}$

1

b.

$$110$$
$$10 \times 11$$
$$2 \times 5 \times 11$$

So, $110 = 2 \times 5 \times 11$.

or

$2\overline{)110} = 2 \times 5 \times 11$

$5\overline{)55}$

$11\overline{)11}$

1

c.

$$120$$
$$10 \times 12$$
$$2 \times 5 \times 2 \times 6$$
$$2 \times 5 \times 2 \times 2 \times 3$$

So, $120 = 2 \times 2 \times 2 \times 3 \times 5$.

or

$2\overline{)120} = 2 \times 2 \times 2 \times 3 \times 5$

$2\overline{)60}$

$2\overline{)30}$

$3\overline{)15}$

$5\overline{)5}$

1

d.

$$360$$
$$10 \times 36$$
$$2 \times 5 \times 6 \times 6$$
$$2 \times 5 \times 2 \times 3 \times 2 \times 3$$

So, $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$.

or

$2\overline{)360} = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$2\overline{)180}$

$2\overline{)90}$

$3\overline{)45}$

$3\overline{)15}$

$5\overline{)5}$

1

10. For question 9.a., the token value of 7 must be added; and for question 9.b., the token value of 11 must be added. For questions 9.c. and 9.d., no new token values need to be added.

Challenge Activity

1. The greatest number of identical submarine sandwiches Sid could have made is 6.

   Cheese: $36 = 2 \times 2 \times 3 \times 3 = 6 \times 6$

   Turkey: $48 = 2 \times 2 \times 2 \times 2 \times 3 = 6 \times 8$

   Tomato: $42 = 2 \times 3 \times 7 = 6 \times 7$

2. Sid put 6 slices of cheese, 8 slices of turkey, and 7 slices of tomato in each sub.

Lesson 2: Representing Patterns

Activity 1

1. **a.** It takes the International Space Station 1.5 h to make one orbit.
   b. The time, in hours, is one and one-half times the number of orbits.

2. **a.** One orbit around the Earth is 43 000 km.
   b. The total distance travelled, in kilometres, is 43 000 times the number of orbits.

3.

Number of Orbits	1	2	3	4	5	6
Time (h)	1.5	3	4.5	6	7.5	9
Distance Travelled (km)	43 000	86 000	129 000	172 000	215 000	258 000

4. The International Space Station makes 16 orbits in one day. From the table, it makes 4 orbits in 6 h. There are 24 h in one day; and since $4 \times 6 \text{ h} = 24 \text{ h}$, $4 \times 4 \text{ orbits} = 16 \text{ orbits}$ per day.

5. Estimates will vary. A sample answer is given.

The space station travels about 688 000 km in one day.

Estimate A

- In question 4, you found that the ISS makes 16 orbits per day.
- From the table, you saw that the distance travelled in one orbit is 43 000 km.
- Estimate $16 \text{ orbits} \times 43\,000 \text{ km}$ per orbit.

$$10 \times 40\,000 \text{ km} = 400\,000 \text{ km and } 20 \times 40\,000 = 800\,000 \text{ km}$$

16 is between 10 and 20, and 688 000 km is between 400 000 km and 800 000 km.

Estimate B

- To estimate $16 \text{ orbits} \times 43\,000 \text{ km}$ per orbit, use compatible numbers and multiply mentally.

$$\begin{aligned} 15 \times 40\,000 &= 15 \times 2 \times 2 \times 10\,000 \\ &= 30 \times 2 \times 10\,000 \\ &= 600\,000 \text{ km} \end{aligned}$$

Using your calculator: $16 \times 43\,000 \text{ km} = 688\,000 \text{ km}$

6. **a.** The crew made 678 orbits while on the International Space Station.

From October 15 at 07:00 UTC to November 26 at 07:00 UTC is 6 weeks.

$$6 \text{ weeks} \times 7 \text{ days per week} = 42 \text{ days}$$

In question 4, you found that there are 16 orbits per day.

$$16 \text{ orbits per day} \times 42 \text{ days} = 672 \text{ orbits}$$

From 07:00 UTC to 16:00 UTC is 9 h. From the table, there are 6 orbits in 9 h.

$$672 \text{ orbits} + 6 \text{ orbits} = 678 \text{ orbits}$$

b. Estimates will vary. A sample answer is given.

The crew travelled about 29 154 000 km while on the International Space Station.

- In question 6.a., you found that the crew made 678 orbits.
- From the table, you saw that the distance travelled in one orbit is 43 000 km.
- Estimate $678 \text{ orbits} \times 43\,000 \text{ km}$.

$$700 \times 40\,000 \text{ km} = 28\,000\,000 \text{ km}$$

Using your calculator: $678 \times 43\,000 \text{ km} = 29\,154\,000 \text{ km}$

7. The shuttle left the International Space Station at 16:30 UTC.

It takes 3 h to make two orbits, and 3 h past 13:30 UTC is 16:30 UTC.

8. Estimates will vary. A sample answer is given.

a. The mission lasted about 10 days.

- Each astronaut was in orbit about 250 h.
- $600 \text{ h} \div 3 = 200 \text{ h}$, and $900 \text{ h} \div 3 = 300 \text{ h}$
- 705 h is between 600 h and 900 h, and 250 h is between 200 h and 300 h.
- 250 h is about 10 days.
- Using your calculator: $705 \text{ h} \div 3 = 235 \text{ h}$ per astronaut
  $235 \text{ h} \div 24$ hours per day $\doteq 9.791\,666\,6$ days

b. Each astronaut completed 156 orbits, and the estimate was about 160 orbits.

- In question 4, you found that there are 16 orbits per day.
- In question 8.a., You found that the mission lasted about 10 days.
- $10 \text{ days} \times 16 \text{ orbits per day} = 160 \text{ orbits}$

Using your calculator: $9.791\,666\,6 \times 16 \text{ orbits per day} \doteq 156.666\,66$

9. The International Space Station makes 1280 orbits in the same time it took Phileas Fogg to go around the world in his hot-air balloon. The space station makes 16 orbits per day, and it took Phileas Fogg 80 days to make one orbit.

$80 \text{ days} \times 16 \text{ orbits per day} = 1280 \text{ orbits}$

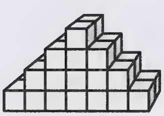
Activity 2

1. Answers will vary. Sample answers are given.

Solution A: To make the fourth building, place a model of the third building on top of a new bottom layer, which would be a 7-by-2 array of blocks.

Solution B: To make the fourth building, beginning with the bottom layer, extend the right side of each layer by adding a 2-by-2 array of 4 blocks. Add a top layer by centering a 1-by-2 array of blocks on the very top.

2. The model of the fourth building is shown below.



3.

Model Number	Number of Layers	Number of Cubes in Bottom Layer	Total Number of Cubes in Model
1	1	2	2
2	2	6	8
3	3	10	18
4	4	14	32

4. The number of layers is the same as the model number.

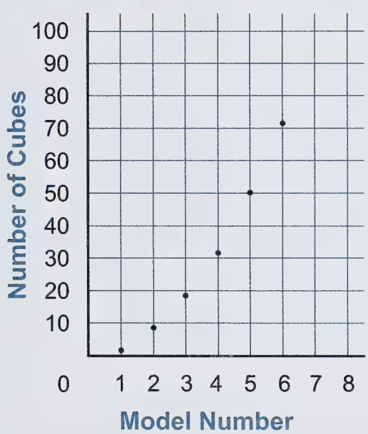
5. There are four more blocks in any given layer than the layer just above it.

6. To get the number of cubes in the bottom layer, you can multiply the model number by 4 and then subtract 2.

7.

Model Number	Number of Cubes in Bottom Layer	Total Number of Cubes in Model
1	$(4\times1)-2=4-2=2$	$(1\times1)\times2=1\times2=2$
2	$(4\times2)-2=8-2=6$	$(2\times2)\times2=4\times2=8$
3	$(4\times3)-2=12-2=10$	$(3\times3)\times2=9\times2=18$
4	$(4\times4)-2=16-2=14$	$(4\times4)\times2=16\times2=32$
5	$(4\times5)-2=20-2=18$	$(5\times5)\times2=25\times2=50$
6	$(4\times6)-2=24-2=22$	$(6\times6)\times2=36\times2=72$

8.



9. a. The number of cubes in the seventh model would be about 100. If you extend the path of dots, the next point should be close to the y value of 100 when it is at the x value of 7.

b. Nanook's method in question 7 said that the total number of cubes in the model is twice as many as the model number multiplied by itself. Therefore, the seventh model would have $(7\times7)\times2=49\times2=98$ cubes.

10. a.

Model Number	Number of Squares on Front Face	Number of Horizontal Roof Squares
1	1	2
2	$1+3=4$	6
3	$1+3+5=9$	10
4	$1+3+5+7=16$	14
5	$1+3+5+7+9=25$	18
6	$1+3+5+7+9+11=36$	22
7	$1+3+5+7+9+11+13=49$	26

b. The number of squares on the front face is the model number multiplied by itself.

c. The number of horizontal roof squares is the same as the number of cubes in the bottom layer.

11. There were 12 Martian buildings in all.

The number of squares on the front face is the model number multiplied by itself. The number you multiply by itself to get 144 is 12 $(12 \times 12 = 144)$.

Activity 3

1. a. Figure 1, Figure 2, and Figure 3 all have one orange square in the centre and four identical arms made with black (right isosceles) triangles. One arm is attached to each side of the square.

b. Each arm in Figure 2 is made with two triangles, which is one more triangle than each arm in Figure 1.

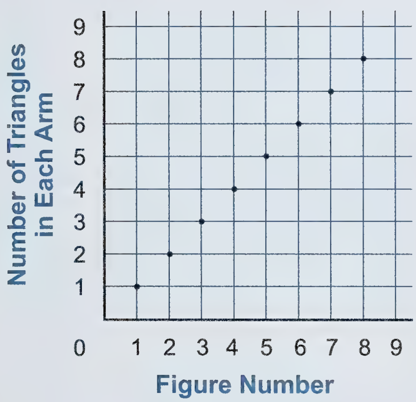
c. Each arm in Figure 3 is made with three triangles, which is one more triangle than each arm in Figure 2.

d. As the pattern grows from one figure to the next, one more triangle is added to each arm.

2. a.

Figure Number	Number of Triangles in Each Arm	Ordered Pair
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)
4	4	(4, 4)
5	5	(5, 5)
6	6	(6, 6)
7	7	(7, 7)
8	8	(8, 8)

b.

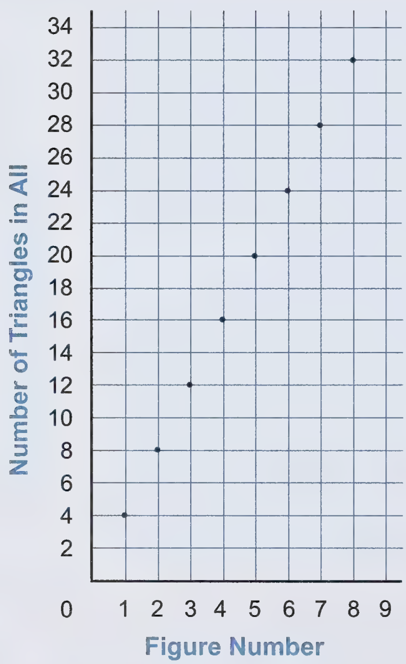


c. The number of triangles in each arm is equal to the figure number.

3. a.

Figure Number	Number of Triangles in All	Ordered Pair
1	4	(1, 4)
2	8	(2, 8)
3	12	(3, 12)
4	16	(4, 16)
5	20	(5, 20)
6	24	(6, 24)
7	28	(7, 28)
8	32	(8, 32)

b.

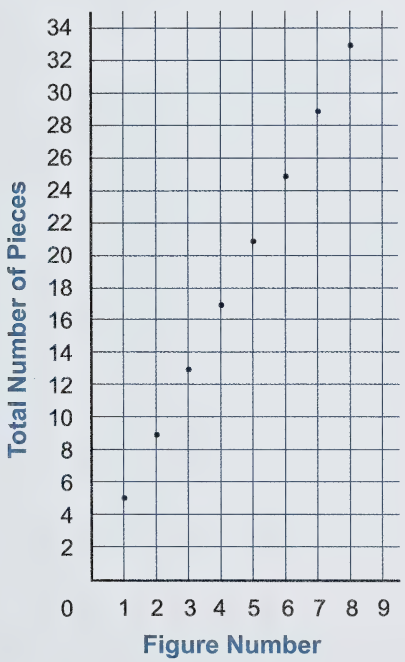


c. The number of triangles in all is 4 times the figure number.

4. a.

Figure Number	Total Number of Pieces	Ordered Pair
1	5	(1, 5)
2	9	(2, 9)
3	13	(3, 13)
4	17	(4, 17)
5	21	(5, 21)
6	25	(6, 25)
7	29	(7, 29)
8	33	(8, 33)

b.



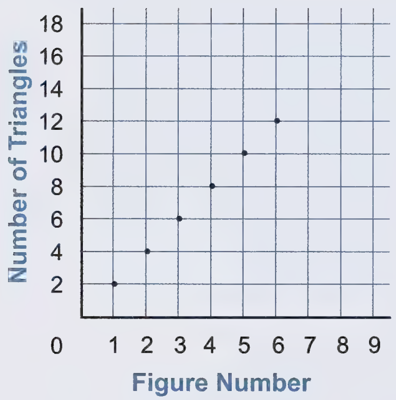
c. To get the total number of pieces, multiply the figure number by 4 and then add 1.

5. As the pattern grows from one figure to the next, the tree grows one square taller and it grows two more triangular leaves.

6. a.

Figure Number	Number of Triangles	Ordered Pair
1	2	(1, 2)
2	4	(2, 4)
3	6	(3, 6)
4	8	(4, 8)
5	10	(5, 10)
6	12	(6, 12)

b.



c. There will be 14 triangles for Figure 7, 16 triangles for Figure 8, and 18 triangles for Figure 9. Extend the straight line path through the dots and read the answers from the graph.

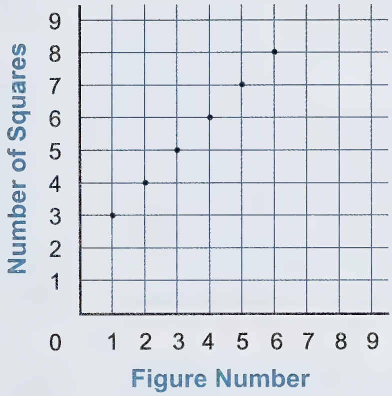
d. The number of triangles is twice the figure number.

e. You will get the same answers using the rule in question 6.d. Two times 7 is 14, 2 times 8 is 16, and 2 times 9 is 18.

7. a.

Figure Number	Number of Squares	Ordered Pair
1	3	(1, 3)
2	4	(2, 4)
3	5	(3, 5)
4	6	(4, 6)
5	7	(5, 7)
6	8	(6, 8)

b.



c. There will be 9 squares for Figure 7, 10 squares for Figure 8, and 11 squares for Figure 9. Extend the straight line path through the dots and read the answers from the graph.

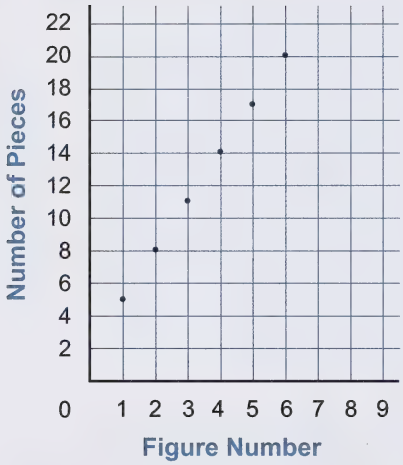
d. The number of squares is 2 more than the figure number.

e. You will get the same answers by using the rule in question 7.d. Two more than 7 is $7 + 2 = 9$; 2 more than 8 is $8 + 2 = 10$; and 2 more than 9 is $9 + 2 = 11$.

8. a.

Figure Number	Total Number of Pieces	Ordered Pair
1	5	$(1, 5)$
2	8	$(2, 8)$
3	11	$(3, 11)$
4	14	$(4, 14)$
5	17	$(5, 17)$
6	20	$(6, 20)$

b.



c. There will be 23 pieces for Figure 7, 26 pieces for Figure 8, and 29 pieces for Figure 9. Extend the straight line path of dots and read the answers from the graph.

d. The total number of pieces is 2 more than 3 times the figure number. So, to find the number of pieces, triple the figure number and then add 2.

e. You will get the same answers by using the rule in question 8.c.

Two more than 3 times 7 is $21+2=23$; 2 more than 3 times 8 is $24+2=26$; and 2 more than 3 times 9 is $27+2=29$.

Challenge Activity

Joshua and Grace sold 30 L of lemonade on July 12.

Trial and error strategy:

- They sold a total of 147 L over 7 days, so that's about 20 L per day.
- Since they sold less at the beginning of the week, begin with 15 L the first day.

Days in Business	Date	Amount Sold (in litres)	
		Guess 1	Guess 2
1	July 6	15	$15 - 3 = 12$
2	July 7	$15 + 3 = 18$	$12 + 3 = 15$
3	July 8	$18 + 3 = 21$	$15 + 3 = 18$
4	July 9	$21 + 3 = 24$	$18 + 3 = 21$
5	July 10	$24 + 3 = 27$	$21 + 3 = 24$
6	July 11	$27 + 3 = 30$	$24 + 3 = 27$
7	July 12	$30 + 3 = 33$	$27 + 3 = 30$
		Total Sold: 168	147

Check: $168 - 147 = 21$ (21 too many) The total was to be 147 L.
Conclusion: $21 \div 7 = 3$ (Make the next guess 3 less.) This is correct.

Lesson 3: Beginning Algebra

Activity 1

1.

Number of Stages	Number of Squares	Number of Trapezoids	Number of Rhombuses	Number of Triangles	Total Number of Blocks
1	3	1	2	1	7
2	5	2	2	1	10
3	7	3	2	1	13
4	9	4	2	1	16

2. a. number of stages: The numbers increase by 1 from model to model.

   b. number of squares: The numbers increase by 2 from model to model.

   c. number of trapezoids: The numbers increase by 1 from model to model.

   d. number of rhombuses: The number doesn't change. It stays equal to 2 for every model.

   e. number of triangles: The number doesn't change. It stays equal to 1 for every model.

   f. total number of blocks: The numbers increase by 3 from model to model.

3. a. The number of trapezoids is the same as the number of stages.

   b. The number of rhombuses is not related to the number of stages. It equals 2, no matter how many stages there are.

   c. The number of triangles is not related to the number of stages. It equals 1, no matter how many stages there are.

   d. The total number of blocks is 4 more than 3 times the number of stages. So, to find the number of blocks, triple the number of stages and add 4.

4. a.

Number of Stages	Number of Squares	Number of Trapezoids	Number of Rhombuses	Number of Triangles	Total Number of Blocks
1	3	1	2	1	7
2	5	2	2	1	10
3	7	3	2	1	13
4	9	4	2	1	16
5	$9+2=11$	$4+1=5$	2	1	$16+3=19$
6	$11+2=13$	$5+1=6$	2	1	$19+3=22$
7	$13+2=15$	$6+1=7$	2	1	$22+3=25$

b. • The number of squares is 1 more than twice the number of stages.

$$\begin{aligned}(7 \times 2) + 1 &= 14 + 1 \\ &= 15 \text{ squares}\end{aligned}$$

• The number of trapezoids is the same as the number of stages = 7 trapezoids.

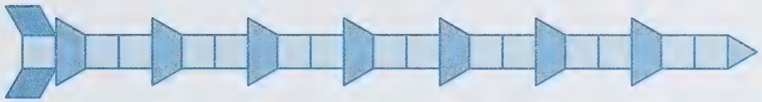
• The number of rhombuses = 2 (always).

• The number of triangles = 1 (always).

• The total number of blocks is 4 more than 3 times the number of stages.

$$\begin{aligned}(7 \times 3) + 4 &= 21 + 4 \\ &= 25 \text{ blocks}\end{aligned}$$

c.



5. a. The method you used in question 4.b. would be faster because you could calculate the number of each shape needed by doing just a few simple arithmetic operations instead of extending the chart and completing down to the fiftieth row.

b. • The number of squares is 1 more than twice the number of stages.

$$\begin{aligned}(50 \times 2) + 1 &= 100 + 1 \\ &= 101 \text{ squares}\end{aligned}$$

• The number of trapezoids is the same as the number of stages = 50 trapezoids.

• The number of rhombuses = 2 (always).

• The number of triangles = 1 (always).

• The total number of blocks is 4 more than 3 times the number of stages.

$$\begin{aligned}(50 \times 3) + 4 &= 150 + 4 \\ &= 154 \text{ blocks}\end{aligned}$$

6. **a.** The number of rhombuses is *n.* This is because the number of rhombuses is the same as the number of stages.

   b. The total number of blocks is $4+(3\times n)$, **or** $(3\times n)+4$, **or** $3n+4$. This is because the total number of blocks is 4 more than 3 times the number of stages.

7. **a.** There are 33 squares in a 16-stage rocket.

   Replace $n=16$ in the expression $1+(n\times 2)$ (or in an equivalent expression).

   $$\begin{aligned}&1+(n\times 2)\\&=1+(16\times 2)\\&=1+32\\&=33 \text{ squares}\end{aligned} \qquad \text{or} \qquad \begin{aligned}&2n+1\\&=(2\times 16)+1\\&=32+1\\&=33 \text{ squares}\end{aligned}$$

   b. There are 2 rhombuses in a 35-stage rocket. In fact, all rockets contain only 2 rhombuses. The number of rhombuses is a constant number (always equal to 2).

   c. There is a total of 67 blocks in a 21-stage rocket.

   Replace $n=21$ in the expression $4+(3\times n)$ (or in an equivalent expression).

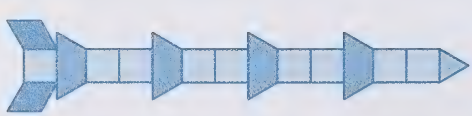
   $$\begin{aligned}&4+(3\times n)\\&=4+(3\times 21)\\&=4+63\\&=67 \text{ blocks}\end{aligned} \qquad \text{or} \qquad \begin{aligned}&3n+4\\&=(3\times 21)+4\\&=63+4\\&=67 \text{ blocks}\end{aligned}$$

8. **a.** There are 4 stages in a rocket model that has 4 rhombuses.

   Represent the number of rhombuses with *r* and the number of stages with *n*.

   $r=n$, so if $r=4$, then $n=4$ stages.

   Verify your solution by drawing a model.



b. There are 13 stages in a rocket model that has 27 squares.

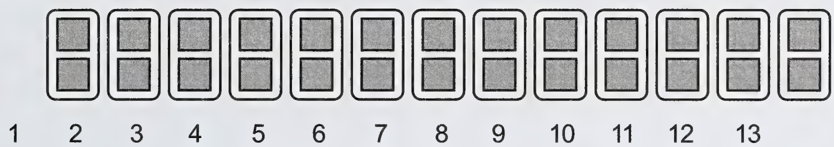
Represent the number of squares with *s* and the number of stages with *n*.

$s = 1 + (n \times 2)$, so if $s = 27$, then $27 = 1 + (n \times 2)$

Begin with 27 square pattern blocks.

Remove 1 square for the base.

Put the remaining squares into pairs.



Count pairs.

There are 13 pairs, so the rocket model has 13 stages.

Verify your solution.

$s = 1 + (n + 2)$

$s = 1 + (13 \times 2)$

$s = 1 + 26$

$s = 27$ squares

c. There are 6 stages in a rocket model that has 22 blocks in total.

Represent the total number of blocks with *b* and the number of stages with *n*.

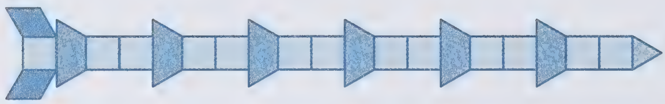
$b = 4 + (3 \times n)$

Count 4 blocks for the base and nose cone, and then skip count 3 more for each stage. Keep track of how many times you need to skip count by 3s to get to 22.

	4	7	10	13	16	19	22
Count by 3s.		+3	+3	+3	+3	+3	+3
Number of skips		1	2	3	4	5	6

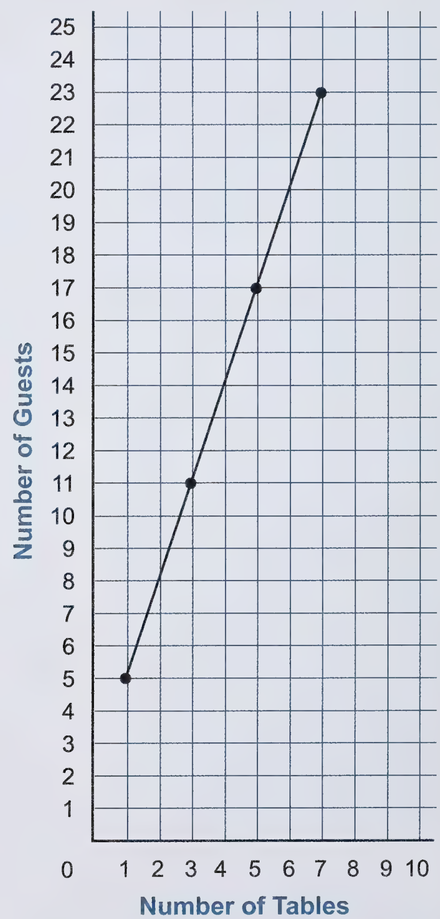
You skip counted 6 times, so the rocket model had 6 stages.

Verify your solution by drawing a model.



Activity 2

1. JoAnn can use her graph to find the number of guests that can be seated at 2, 4, and 6 tables by joining the points with a line segment and reading the values.



2. a.

Number of Tables (t)	Number of Guests (g)
1	5
2	8
3	11
4	14
5	17
6	20
7	23

b. Each time another table is added, three more guests can be seated.

c.

Number of Tables (t)	Number of Guests (g)
8	$23+3=26$
9	$26+3=29$
10	$29+3=32$

3. a. The number of guests that can be seated is 2 more than 3 times the number of tables.

b. $g=3\times t+2$

4. a. For 8 tables, $g=3\times t+2$

$$\begin{aligned} &=(3\times 8)+2\\ &=24+2\\ &=26 \text{ guests}\end{aligned}$$

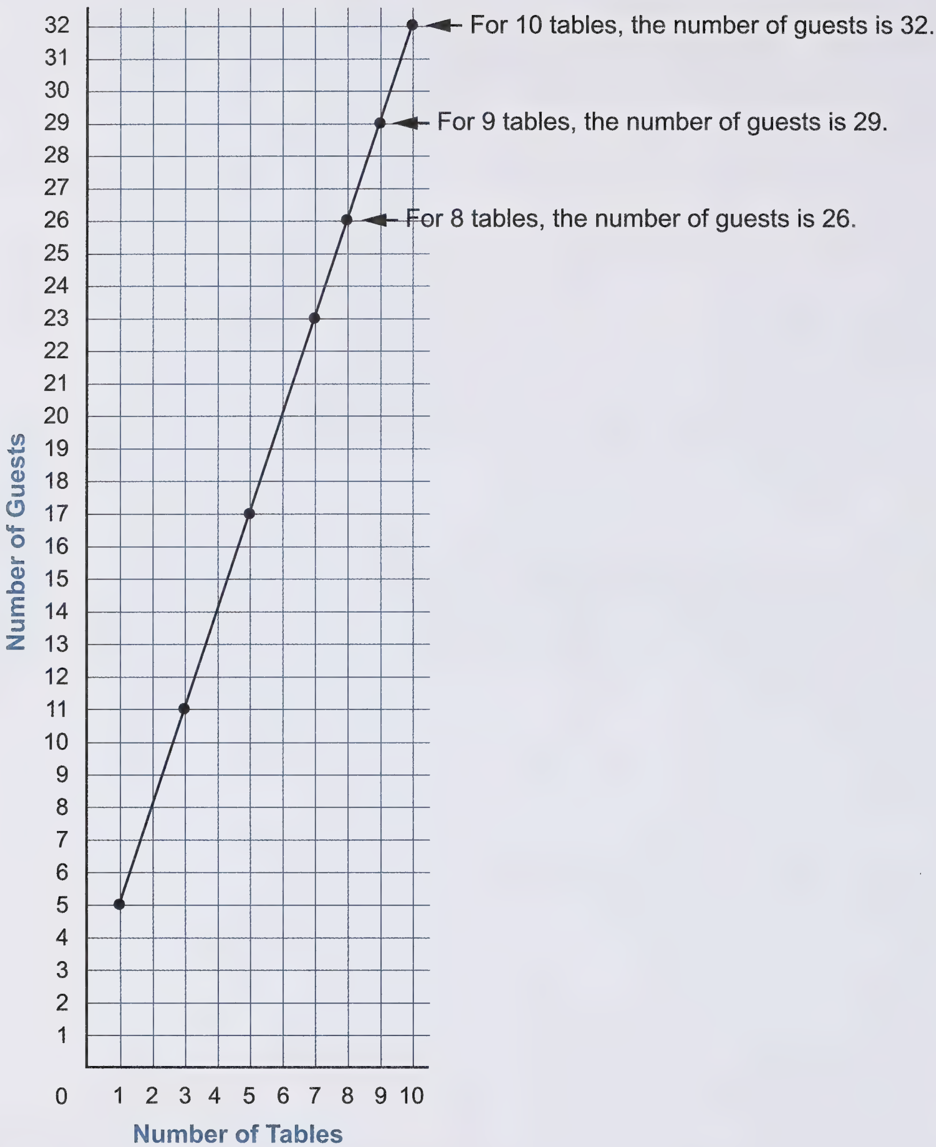
For 9 tables, $g=3\times t+2$

$$\begin{aligned} &=(3\times 9)+2\\ &=27+2\\ &=29 \text{ guests}\end{aligned}$$

For 10 tables, $g=3\times t+2$

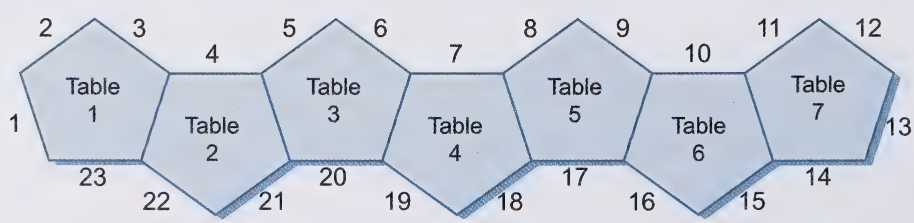
$$\begin{aligned} &=(3\times 10)+2\\ &=30+2\\ &=32 \text{ guests}\end{aligned}$$

b.



5. a. The tables are pentagonal in shape.

b.

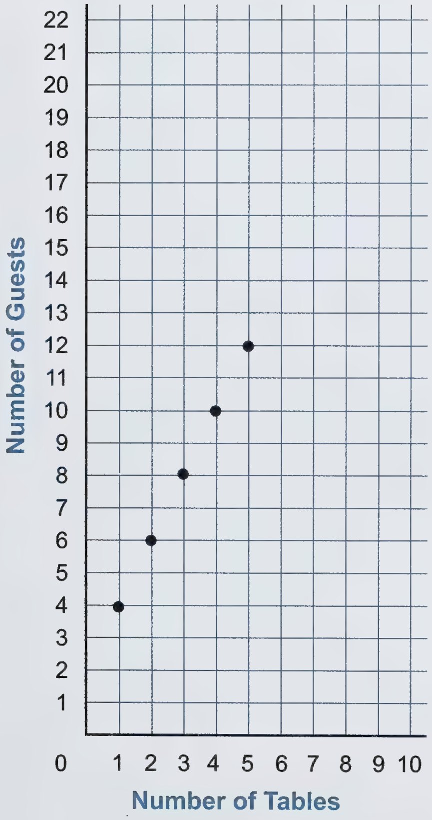


6. a.

Number of Tables (t)	Number of Guests $g = 2t + 2$
1	4
2	6
3	8
4	10
5	12

b. For each table that is added, two additional guests can be seated.

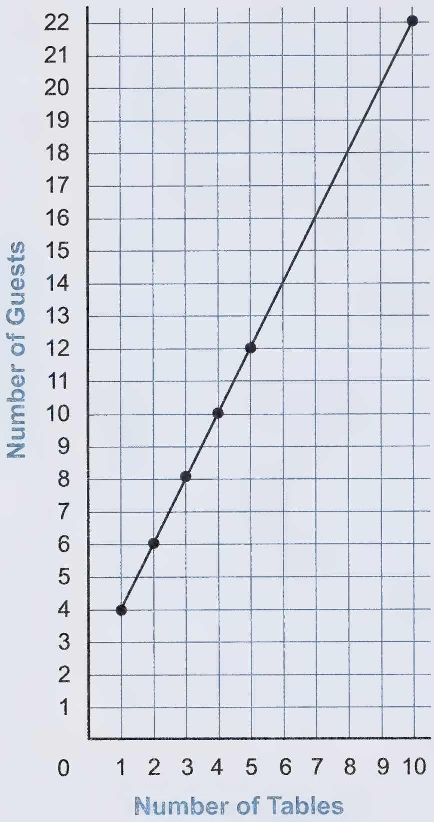
c.



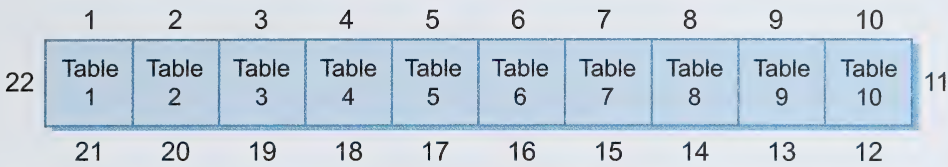
d. For 10 tables, $g = 2 \times t + 2$

$$\begin{aligned} &= (2 \times 10) + 2 \\ &= 20 + 2 \\ &= 22 \end{aligned}$$

e.



f.



7. a.

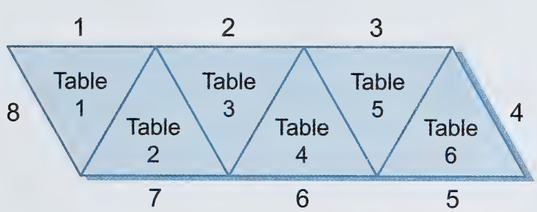
Number of Tables	Number of Guests
1	6
2	10
3	14
4	18
5	22

b. Let the number of guests that can be seated $= g$.

Let the number of tables used $= t$.

$$g = 4 \times t + 2$$

8. a. Eight guests can be seated around 6 tables.

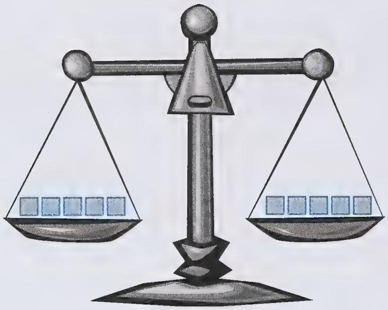


b. $g = t + 2$

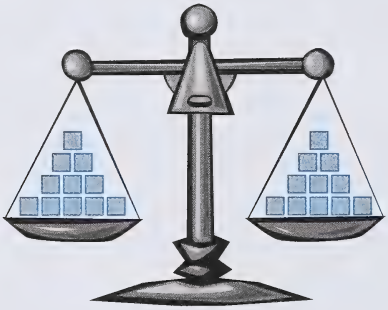
9. In all the equations, you multiply the number of tables by 2 less than the number of sides on each table and then add 2. When two tables are joined along an edge, two of the edges are on the inside of the longer table.

Activity 3

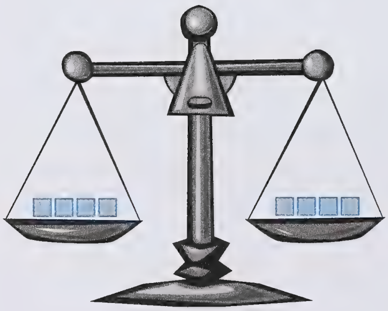
1. a. The scale will be balanced because there is the same number of blocks (5) on each pan.



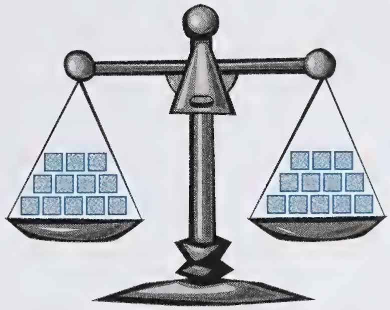
b. The scale will stay balanced because there is still the same number of blocks (11) on each pan.



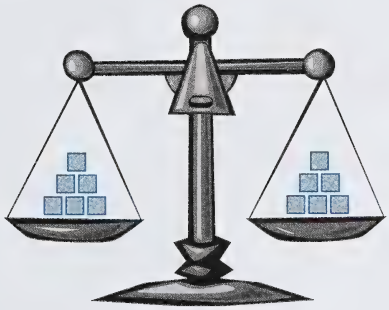
c. The scale will stay balanced because there is still the same number of blocks (4) on each pan.



d. The scale will stay balanced because there is still the same number of blocks (12) on each pan.



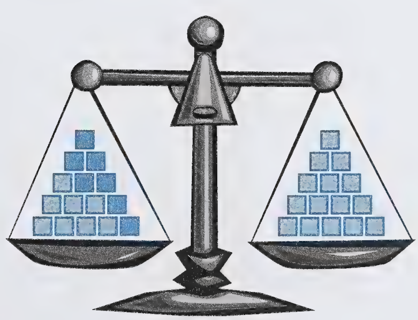
e. The scale will stay balanced because there is still the same number of blocks (6) on each pan.



2. a. 8 + 7 = 15

Put 8 blocks in the left pan and 15 blocks in the right pan.

Add 7 blocks to the left pan to balance the scale.

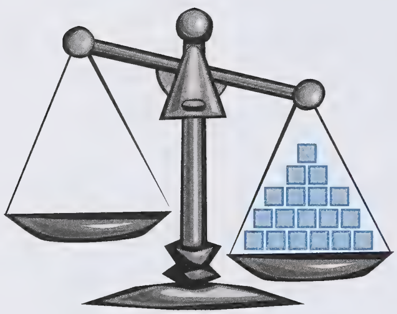


Work backwards. To find the number you must add to 8 to get 15, you can subtract.

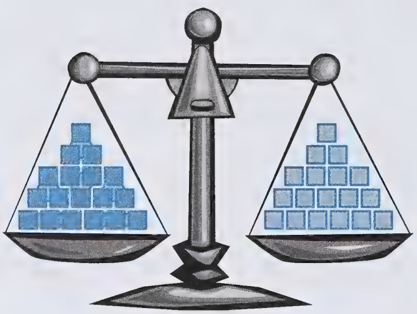
$15 - 8 = 7$

b. $\boxed{6} \times 3 = 18$

Put 18 blocks in the right pan.



Add 6 sets of 3 blocks each to the left pan to balance the scale.

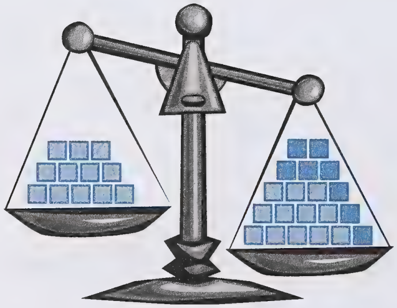


Work backwards. To find the number you must multiply by 3 to get 18, you can divide.

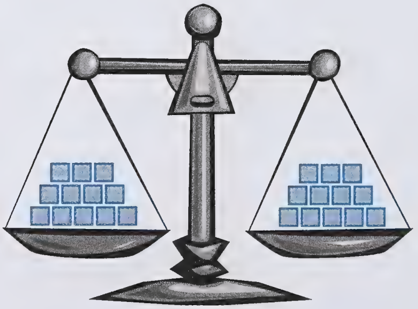
$18 \div 3 = 6$

c. $12 = 20 - \boxed{8}$

Put 12 blocks in the left pan and 20 blocks in the right pan.



Remove 8 blocks from the right pan to balance the scale.

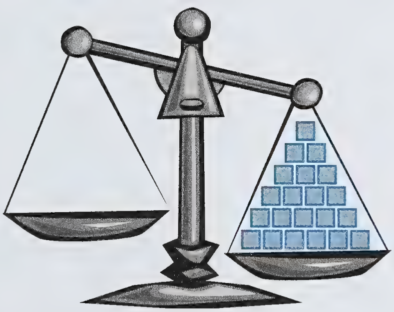


Work backwards. To find the number you must subtract from 20 to get 12, think about what number you can add to 12 to total 20. To get that you can subtract.

$20 - 12 = 8$

d. $7 = 21 \div 3$

Put 21 blocks in the right pan.



Remove 3 sets of 7 blocks each from the right pan to balance the scale.

Work backwards. To find the number you must divide 21 by to get 7. Think about what number you can multiply 7 by to get 21. To get that, you can divide.

$21 \div 7 = 3$

3. Textbook, page 86, questions 1 to 4

1. The number is 18.

a. Solution (working backwards)

- Begin with 21.
- Add 15.

  $21 + 15 = 36$

- Divide by 2.

  $36 \div 2 = 18$

b. Check (working forwards)

- Begin with 18.
- Multiply by 2.

  $18 \times 2 = 36$

- Subtract 15.

  $36 - 15 = 21$

2. The number is 125.

a. Solution (working backwards)

- Begin with 75.
- Multiply by 2.

$75 \times 2 = 150$

- Subtract 25.

$150 - 25 = 125$

b. Check (working forwards)

- Begin with 125.
- Add 25.

$125 + 25 = 150$

- Divide by 2.

$150 \div 2 = 75$

3. The number is 4.

a. Solution (working backwards)

- Begin with 1.
- Multiply by 100.

$1 \times 100 = 100$

- Divide by 5.

$100 \div 5 = 20$

- Subtract 16.

$20 - 16 = 4$

b. Check (working forwards)

- Begin with 4.
- Add 16.

$4 + 16 = 20$

- Multiply by 5.

$5 \times 20 = 100$

- Divide by 100.

$100 \div 100 = 1$

4. The number is 2.

a. Solution (working backwards)

- Begin with 35.
- Subtract 2.

$$35 - 2 = 33$$

- Subtract 25.

$$33 - 25 = 8$$

- Divide by 4.

$$8 \div 4 = 2$$

b. Check (working forwards)

- Begin with 2.
- Multiply by 4.

$$2 \times 4 = 8$$

- Add 25.

$$8 + 25 = 33 = 35 - 2$$

4. a. Textbook, page 86, question 5

- Begin with 3.
- Multiply by 2.

$$2 \times 3 = 6$$

- Multiply by 2.

$$2 \times 6 = 12$$

- Divide by 4.

$$12 \div 4 = 3$$

- Begin with 5.
- Multiply by 2.

$$2 \times 5 = 10$$

- Multiply by 2.

$$10 \times 2 = 20$$

- Divide by 4.

$$20 \div 4 = 5$$

- Begin with 7.
- Multiply by 2.

  $2 \times 7 = 14$

- Multiply by 2.

  $2 \times 14 = 28$

- Divide by 4.

  $28 \div 4 = 7$

- Begin with 10.
- Multiply by 2.

  $2 \times 10 = 20$

- Multiply by 2.

  $2 \times 20 = 40$

- Divide by 4.

  $40 \div 4 = 10$

b. The quotient at the end will always equal the starting number. Doubling a number twice in a row is the same as multiplying the number by 4. If you then divide the answer by 4, you always get back to the starting number. This is because the division "undoes" what the multiplication did.

Challenge Activity

The total cost for the 72 balls will be $600.

Type of Ball	Number Needed		Cost Per Ball	Total Cost
Footballs	12	×	$25.00	$ 300.00
Soccerballs	$12 \div 2 = 6$	×	$20.00	$ 120.00
Baseballs	$3 \times 6 = 18$	×	$5.00	$ 90.00
Tennis Balls	$2 \times 18 = 36$	×	$2.50	$ 90.00
	Total: 72 balls			$600.00

Keystrokes

1. a.

Keystrokes	ON/C	247	×	7	×	11	×	13	=
Display	0	247	247	7	1729	11	19019	13	247247

   b. The result is a six-digit number formed by repeating the digits 247 twice.

   c. Answers will vary. Sample answers are given.

Keystrokes	ON/C	543	×	7	×	11	×	13	=
Display	0	543	543	7	3801	11	41811	13	543543

Keystrokes	ON/C	111	×	7	×	11	×	13	=
Display	0	111	111	7	777	11	8547	13	111111

Keystrokes	ON/C	404	×	7	×	11	×	13	=
Display	0	404	404	7	2828	11	31108	13	404404

   d. You could multiply your three-digit number by 1001 to get the same results. That is because $7 \times 11 \times 13 = 1001$. Multiplying a number by 1001 is the same as multiplying the number by 1000 and then adding the number.

2. a. If you enter a six-digit number that has the same pattern as your resulting numbers in question 1, then divide by 7, then divide by 11, and then divide by 13, the answer will be a three-digit number that was repeated to make the starting number.

   b. Answers will vary. Sample answers are given.

Keystrokes	ON/C	263263	÷	7	÷	11	÷	13	=
Display	0	263263	263263	7	37609	11	3419	13	263

Keystrokes	ON/C	999999	÷	7	÷	11	÷	13	=
Display	0	999999	999999	7	142857	11	12987	13	999

3. Answers will vary. A sample answer is given.

Use the birthday April 23.

	Keystrokes	Display
Enter the number of the month you were born in. $(\text{Jan.} = 1 \text{ Feb.} = 2, \ldots, \text{Dec.} = 12)$	4	4
Multiply by 5.	×	4
	5	5
Add 5.	+	20
	5	5
Multiply by 20.	×	25
	20	20
Add the day of the month on which you were born. $(1\text{st} = 1,\ 2\text{nd} = 2, \ldots,\ 31\text{st} = 31)$	+	500
	23	23
Subtract 100.	−	523
	100	100
	=	423

4. Work backwards.

	Keystrokes	Display
Enter the final number on the display in question 1.	423	423
Add 100.	+	423
	100	100
Subtract the day of the month.	−	523
	23	23
Divide by 20.	÷	500
	20	20
Subtract 5.	−	25
	5	5
Divide by 5.	÷	20
	5	5
	=	4

Review

1. a. **Textbook, page 67, On Your Own, questions 1 and 2**

   1. George, Manuel, and Jonathan will all visit England again in 1992.

1980	81	82	83	84	85	86	87	88	89	90	91	92
George		G		G		G		G		G		G
Manuel			M			M			M			M
Jonathan				J				J				J

   2. Karen and Mark will work together again in 12 days.

Today	1	2	3	4	5	6	7	8	9	10	11	12
Mark				M				M				M
Karen						K						K

2. **Textbook, page 67, Practise Your Skills, questions 1 to 8**

   1. multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40

      multiples of 8: 8, 16, 24, 32, 40

      multiples of 40: 40, 80, 120, 160, 200

   2. multiples of 2: 2, 4, 6, 8, 10

      multiples of 5: 5, 10

      multiples of 10: 10, 20, 30, 40, 50

3. multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36

   multiples of 9: 9, 18, 27, 36

   multiples of 36: 36, 72, 108, 144, 180

4. multiples of 6: 6, 12, 18, 24, 30

   multiples of 10: 10, 20, 30

   multiples of 30: 30, 60, 90, 120, 150

5. multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48

   multiples of 8: 8, 16, 24, 32, 40, 48

   multiples of 48: 48, 96, 144, 192, 240

6. multiples of 3: 3, 6, 9, 12, 15

   multiples of 5: 5, 10, 15

   multiples of 15: 15, 30, 45, 60, 75

7. multiples of 2: 2, 4, 6, 8

   multiples of 8: 8

   multiples of 8: 8, 16, 24, 32, 40

8. multiples of 4: 4, 8, 12

   multiples of 6: 6, 12

   multiples of 12: 12, 24, 36, 48, 60

3. **Textbook, page 96, Skill Bank from This Unit, question 1**

   1. **a.** multiples of 3: 3, 6, 9, 12

      multiples of 4: 4, 8, 12

      LCM = 12

      b. multiples of 5: 5, 10, 15, 20, 25, 30, 35

      multiples of 7: 7, 14, 21, 28, 35

      LCM = 35

      c. multiples of 6: 6, 12, 18, 24 , 30, 36, 42, 48

      multiples of 8: 8, 16, 24 , 32, 40, 48

      LCM = 24

      d. multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34

      multiples of 17: 17, 34

      LCM = 34

4. **Textbook, page 123, Skill Bank Looking Back, question 6**

   6. **a.** multiples of 2: 2, 4, 6

      multiples of 3: 3, 6

      LCM = 6

      b. multiples of 4: 4, 8, 16, 20

      multiples of 5: 5, 10, 15, 20

      LCM = 20

c. multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18

multiples of 9: 9, 18

LCM = 18

5. Textbook, page 68, Exploring Factors

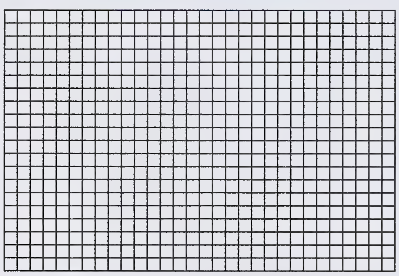
First, find the common factors of 20 and 30.

$20 = 2 \times 10$ or 4×5, so its factors are 1, 2, 4, 5, 10, and 20.

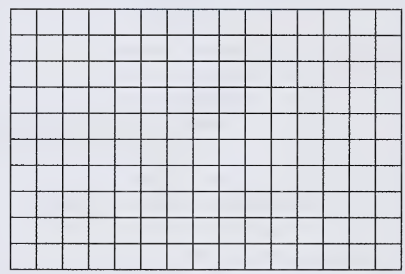
$30 = 2 \times 15, 3 \times 10$, or 5×6, so its factors are 1, 2, 3, 5, 6, 10, 15, and 30.

Mary can divide her garden into congruent squares in the following four ways.

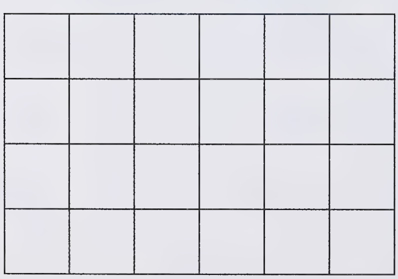
1 m by 1 m squares



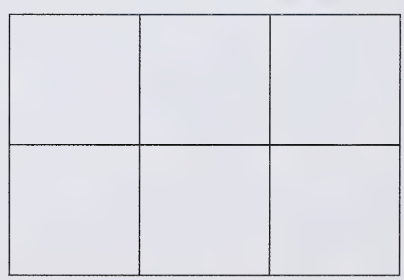
2 m by 2 m squares



5 m by 5 m squares



10 m by 10 m squares



6. **Textbook, page 70, Practise Your Skills, question 1**

1. **a.** $40 = 1 \times 40$, 2×20, 4×10, or 5×8, so its factors are 1, 2, 4, 5, 8, 10, 20, and 40.

   $56 = 1 \times 56$, 2×28, 4×14, or 7×8, so its factors are 1, 2, 4, 7, 8, 14, 28, and 56.

   The GCF of 40 and 56 is 8.

   b. $24 = 1 \times 24$, 2×12, 3×8, or 4×6, so its factors are 1, 2, 3, 4, 6, 8, 12, and 24.

   $40 = 1 \times 40$, 2×20, 4×10, or 5×8, so its factors are 1, 2, 4, 5, 8, 10, 20, and 40.

   The GCF of 24 and 40 is 8.

   c. $16 = 1 \times 16$, 2×8, or 4×4, so its factors are 1, 2, 4, 8, and 16.

   $32 = 1 \times 32$, 2×16, or 4×8, so its factors are 1, 2, 4, 8, 16, and 32.

   The GCF of 16 and 32 is 16

   d. $30 = 1 \times 30$, 2×15, 3×10, or 5×6, so its factors are 1, 2, 3, 5, 6, 10, 15, and 30.

   $32 = 1 \times 32$, 2×16, or 4×8, so its factors are 1, 2, 4, 8, 16, and 32.

   The GCF of 30 and 32 is 2.

7. **Textbook, page 96, Skill Bank from This Unit, question 2 and 3**

2. **a.** $54 = 1 \times 54$, 2×27, 3×18, or 6×9, so its factors are 1, 2, 3, 6, 9, 18, 27, and 54.

   b. $27 = 1 \times 27$ or 3×9, so its factors are 1, 3, 9, and 27.

c. $40 = 1 \times 40$, 2×20, 4×10, or 5×8, so its factors are 1, 2, 4, 5, 8, 10, 20, and 40.

d. $18 = 1 \times 18$, 2×9, or 3×6, so its factors are 1, 2, 3, 6, 9, and 18.

e. $17 = 1 \times 17$, so its factors are 1 and 17.

f. $36 = 1 \times 36$, 2×18, 3×12, 4×9, or 6×6, so its factors are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

3. a. $16 = 1 \times 16$, 2×8, or 4×4, so its factors are 1, 2, 4, 8, and 16.

$24 = 1 \times 24$, 2×12, 3×8, or 4×6, so its factors are 1, 2, 3, 4, 6, 8, 12, and 24.

The GCF of 16 and 24 is 8.

b. $12 = 1 \times 12$, 2×6, or 3×4, so its factors are 1, 2, 3, 4, 6, and 12.

$32 = 1 \times 32$, 2×16, or 4×8, so its factors are 1, 2, 4, 8, 16, and 32.

The GCF of 12 and 32 is 4.

c. $24 = 1 \times 24$, 2×12, 3×8, or 4×6, so its factors are 1, 2, 3, 4, 6, 8, 12, and 24.

$30 = 1 \times 30$, 2×15, 3×10, or 5×6, so its factors are 1, 2, 3, 5, 6, 10, 15, and 32.

The GCF of 24 and 30 is 6.

d. $36 = 1 \times 36$, 2×18, 3×12, 4×9, or 6×6, so its factors are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

$40 = 1 \times 40$, 2×20, 4×10, or 5×8, so its factors are 1, 2, 4, 5, 8, 10, 20, and 40.

The GCF of 36 and 40 is 4.

8. **Textbook, page 123, Skill Bank Looking Back, question 7**

7. a. $9 = 1 \times 9$ or 3×3, so its factors are 1, 3, and 9.

$12 = 1 \times 12$, 2×6, or 3×4, so its factors are 1, 2, 3, 4, 6, and 12.

The GCF of 9 and 12 is 3.

b. $15 = 1 \times 15$ or 3×5, so its factors are 1, 3, 5, and 15.

$20 = 1 \times 20$, 2×10, or 4×5, so its factors are 1, 2, 4, 5, 10, and 20.

The GCF of 15 and 20 is 5.

c. $16 = 1 \times 16$, 2×8, or 4×4, so its factors are 1, 2, 4, 8, and 16.

$24 = 1 \times 24$, 2×12, 3×8, or 4×6, so its factors are 1, 2, 3, 4, 6, 8, 12, and 20.

The GCF of 16 and 24 is 8.

9. **Textbook, page 145, Skill Bank Looking Back, question 1**

1. a. $28 = 1 \times 28$, 2×14, or 4×7, so its factors are 1, 2, 4, 7, 14, and 28.

b. $24 = 1 \times 24$, 2×12, 3×8, or 4×6, so its factors are 1, 2, 3, 4, 6, 8, 12, and 24.

c. $15 = 1 \times 15$ or 3×5, so its factors are 1, 3, 5, and 15.

d. $50 = 1 \times 50$, 2×25, or 5×10, so its factors are 1, 2, 5, 10, 25, and 50.

10. Textbook, page 70, Practise Your Skills, question 2

2. a. 57 composite A factor pair is 3×29.
   b. 19 prime
   c. 91 composite A factor pair is 7×13.
   d. 100 composite Factor pairs are 2×50, 4×25, 5×20, and 10×10.
   e. 69 composite A factor pair is 3×23.
   f. 12 composite Factor pairs are 2×6 and 3×4.
   g. 37 prime
   h. 81 composite Factor pairs are 3×27 and 9×9.
   i. 46 composite A factor pair is 2×23.

11. Textbook, page 96, Skill Bank from This Unit, question 4

4. Answers will vary. Sample answers are given.

   Prime numbers less than 100: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

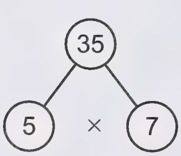
   Composite numbers less than 100: any even number greater than 2, along with 9, 15, 21, 25, 27, 33, 35, 39, 45, 49, 51, 55, 57, 63, 65, 69, 75, 77, 81, 85, 87, 91, 93, 95, and 99

12. Textbook, page 145, Skill Bank Looking Back, question 2

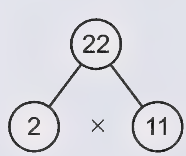
2.
- 21 composite
- 22 composite
- 23 prime
- 24 composite
- 25 composite
- 26 composite
- 27 composite
- 28 composite
- 29 prime
- 30 composite

13. Textbook, pages 69 to 70, On Your Own, questions 1 and 2

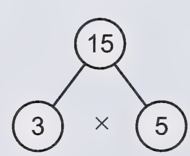
1. a.



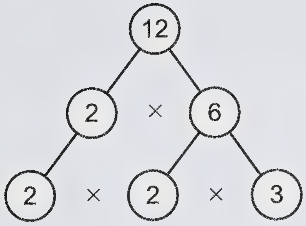
b.



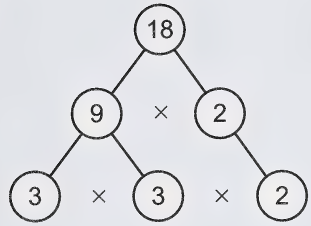
c.



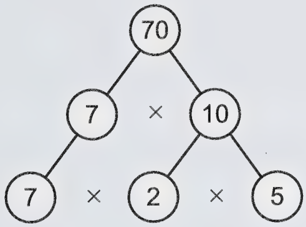
d.



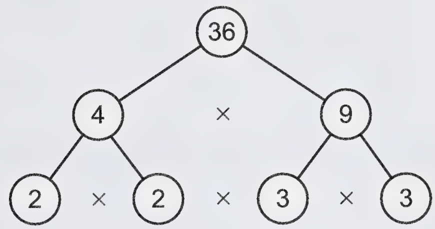
e.



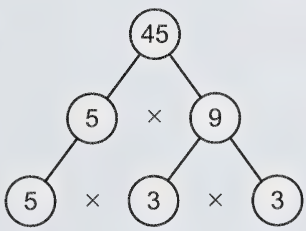
f.



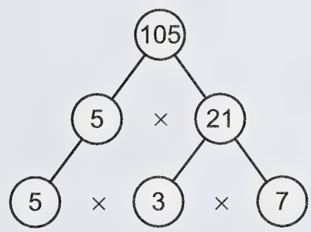
g.



h.



i.



2. a. $2\overline{)96} = 2\times2\times2\times2\times2\times3$

$2\overline{)48}$

$2\overline{)24}$

$2\overline{)12}$

$2\overline{)6}$

3

b. $2\overline{)100} = 2\times2\times5\times5$

$2\overline{)50}$

$5\overline{)25}$

5

c. $2\overline{)72} = 2\times2\times2\times3\times3$

$2\overline{)36}$

$2\overline{)18}$

$3\overline{)9}$

3

d. $2\overline{)120} = 2\times2\times2\times3\times5$

$2\overline{)60}$

$2\overline{)30}$

$3\overline{)15}$

5

e. $2\overline{)200} = 2\times2\times2\times5\times5$

$2\overline{)100}$

$2\overline{)50}$

$5\overline{)25}$

5

f. $2\overline{)64} = 2\times2\times2\times2\times2\times2$

$2\overline{)32}$

$2\overline{)16}$

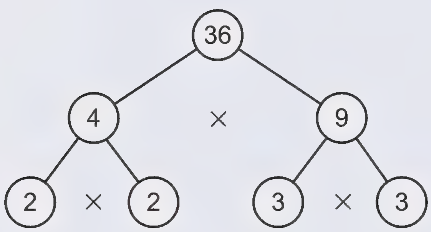
$2\overline{)8}$

$2\overline{)4}$

2

14. Textbook, page 145, Skill Bank Looking Back, question 3

3. a.



and

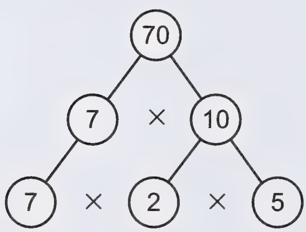
$2\overline{)36} = 2\times2\times3\times3$

$2\overline{)18}$

$3\overline{)9}$

3

b.



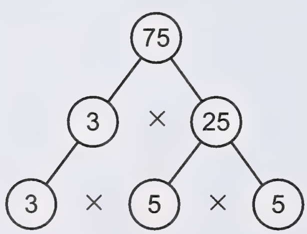
and

$2\overline{)70} = 2\times5\times7$

$5\overline{)35}$

7

c.



and

$3\overline{)75} = 3\times5\times5$

$5\overline{)25}$

5

15. a. The sun rises 1 min later one day, followed by 2 min later the next day. This alternating pattern appears to continue through September.

b. The sun sets 2 min earlier for two days in a row, and on the third day it sets 1 min earlier. This three-day pattern appears to continue through September.

c.

Date	Sunrise Time (A.M.)
Sept 17	7:21
Sept 18	7:23
Sept 19	7:24
Sept 20	7:26

Date	Sunset Time (P.M.)
Sept 19	7:51
Sept 20	7:49
Sept 21	7:47
Sept 22	7:46

16. Textbook, page 26, Finding Rules, questions 1 to 4

1. Multiply the first number by the second, and then subtract 1. The number sets 10, 2, 19 and 4, 6, 23 follow this rule.

2. Multiply the first number by the second, and then add 4. The number sets 3, 0, 4 and 7, 1, 11 follow this rule.

3. Divide the first number by the second, and then add 1. The number sets 16, 4, 5 and 18, 6, 4 follow this rule.

4. Divide the first number by the second, and then subtract 1. The number sets 20, 4, 4 and 17, 1, 16 follow this rule

17. Textbook, page 27, On Your Own, questions 1 to 4 and 6 to 8

1. Multiply the first number by the second.

2. Add the first and second numbers.

3. Multiply the first number by the second, and then subtract 1.

4. Divide the first number by the second.

6. Add the first and second numbers, and then multiply the sum by 2.

7. Subtract the second number from the first, and then add 2.

8. Think about which operation(s) can be used with the first two numbers to equal the third number. It helps to keep in mind the relative size of the numbers and concepts like factors and multiples.

18. Textbook, page 15, Practise Your Skills, questions 1 to 2

1. Multiply by 3.

Input	Output
0	0
3	9
7	21
10	30
12	36
15	45
20	60

2. Add 4.

Input	Output
0	4
8	12
11	15
14	18
20	24
35	39
45	49

19. Textbook, page 21, Practise Your Skills, questions 1 to 3

1. Multiply by 2, and then add 1.

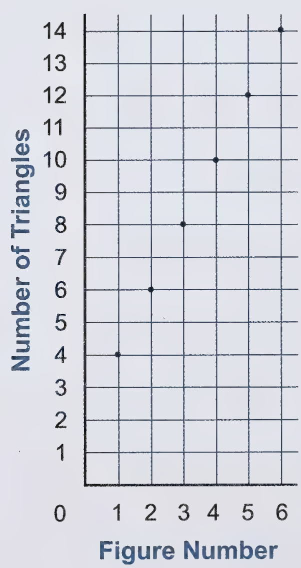
2. Multiply by 4.

3. Multiply the number by itself.

20. a.

Figure Number	Number of Triangles
1	4
2	6
3	8
4	10
5	12
6	14

b.



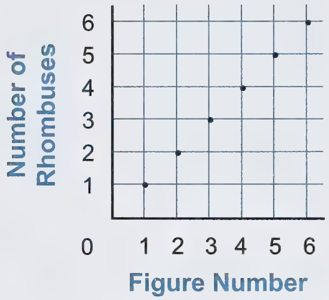
c. The number of triangles (t) is two more than twice the figure number (n).

$t = (2 \times n) + 2$

21. a.

Figure Number	Number of Rhombuses
1	1
2	2
3	3
4	4
5	5
6	6

b.



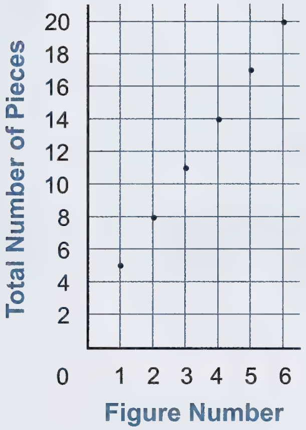
c. The number of rhombuses (r) is equal to the figure number (n).

$r = n$

22. a.

Figure Number	Total Number of Pieces
1	5
2	8
3	11
4	14
5	17
6	20

b.



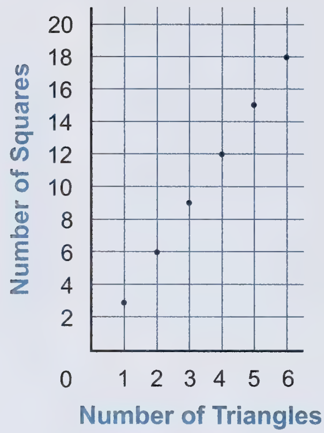
c. The total number of pieces (p) is two more than three times the figure number (n).

$p = (3 \times n) + 2$

23. Textbook, page 18, On Your Own, questions 1 and 2

1.

Triangles	Squares
1	3
2	6
3	9
4	12
5	15
6	18



2. You would need 45 squares for 15 triangles because the number of squares (s) is 3 times the number of triangles (t).

$$s = 3 \times t$$

24. Textbook, page 89, On Your Own, questions 1 to 4

Answers will vary. All possible answers are given.

1.

1-g	5-g	10-g	Left side	= Right side	= Total
25			$25 \times 1\text{-g}$	$= (2 \times 10\text{-g}) + (1 \times 5\text{-g})$	$= 25\text{g}$
20	1		$(20 \times 1\text{-g}) + (1 \times 5\text{-g})$	$= (2 \times 10\text{-g}) + (1 \times 5\text{-g})$	$= 25\text{g}$
15	2		$(15 \times 1\text{-g}) + (2 \times 5\text{-g})$	$= (2 \times 10\text{-g}) + (1 \times 5\text{-g})$	$= 25\text{g}$
15		1	$(15 \times 1\text{-g}) + (1 \times 10\text{-g})$	$= (2 \times 10\text{-g}) + (1 \times 5\text{-g})$	$= 25\text{g}$
10	3		$(10 \times 1\text{-g}) + (3 \times 5\text{-g})$	$= (2 \times 10\text{-g}) + (1 \times 5\text{-g})$	$= 25\text{g}$
10	1	1	$(10 \times 1\text{-g}) + (1 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= (2 \times 10\text{-g}) + (1 \times 5\text{-g})$	$= 25\text{g}$
5	4		$(5 \times 1\text{-g}) + (4 \times 5\text{-g})$	$= (2 \times 10\text{-g}) + (1 \times 5\text{-g})$	$= 25\text{g}$

5	2	1	$(5\times 1\text{-g})+(2\times 5\text{-g})+(1\times 10\text{-g})$	$=(2\times 10\text{-g})+(1\times 5\text{-g})$	$=25\,g$
5		2	$(5\times 1\text{-g})+(2\times 10\text{-g})$	$=(2\times 10\text{-g})+(1\times 5\text{-g})$	$=25\,g$
	5		$5\times 5\text{-g}$	$=(2\times 10\text{-g})+(1\times 5\text{-g})$	$=25\,g$
	3	1	$(3\times 5\text{-g})+(1\times 10\text{-g})$	$=(2\times 10\text{-g})+(1\times 5\text{-g})$	$=25\,g$
	1	2	$(1\times 5\text{-g})+(2\times 10\text{-g})$	$=(2\times 10\text{-g})+(1\times 5\text{-g})$	$=25\,g$

2.

1-g	5-g	10-g	Left side	= Right side	= Total
11			$11\times 1\text{-g}$	$=11\times 1\text{-g}$	$=11\,g$
6	1		$(6\times 1\text{-g})+(1\times 5\text{-g})$	$=11\times 1\text{-g}$	$=11\,g$
1	2		$(1\times 1\text{-g})+(2\times 5\text{-g})$	$=11\times 1\text{-g}$	$=11\,g$
1		1	$(1\times 1\text{-g})+(1\times 10\text{-g})$	$=11\times 1\text{-g}$	$=11\,g$

3.

1-g	5-g	10-g	Left side	= Right side	= Total
15			$15\times 1\text{-g}$	$=(5\times 1\text{-g})+(1\times 10\text{-g})$	$=15\,g$
10	1		$(10\times 1\text{-g})+(1\times 5\text{-g})$	$=(5\times 1\text{-g})+(1\times 10\text{-g})$	$=15\,g$
5	2		$(5\times 1\text{-g})+(2\times 5\text{-g})$	$=(5\times 1\text{-g})+(1\times 10\text{-g})$	$=15\,g$
5		1	$(5\times 1\text{-g})+(1\times 10\text{-g})$	$=(5\times 1\text{-g})+(1\times 10\text{-g})$	$=15\,g$
	3		$3\times 5\text{-g}$	$=(5\times 1\text{-g})+(1\times 10\text{-g})$	$=15\,g$
	1	1	$(1\times 5\text{-g})+(1\times 10\text{-g})$	$=(5\times 1\text{-g})+(1\times 10\text{-g})$	$=15\,g$

4.

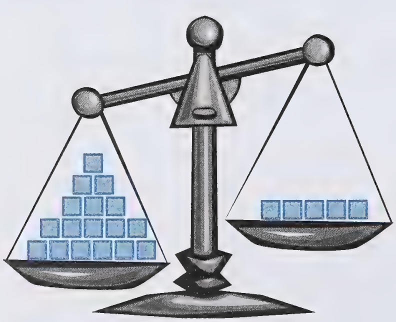
1-g	5-g	10-g	Left side	= Right side	= Total
20			$20 \times 1\text{-g}$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
15	1		$(15 \times 1\text{-g}) + (1 \times 5\text{-g})$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
10	2		$(10 \times 1\text{-g}) + (2 \times 5\text{-g})$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
10		1	$(10 \times 1\text{-g}) + (1 \times 10\text{-g})$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
5	3		$(5 \times 1\text{-g}) + (3 \times 5\text{-g})$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
5	1	1	$(5 \times 1\text{-g}) + (1 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
	4		$4 \times 5\text{-g}$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
	2	1	$(2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$
		2	$2 \times 10\text{-g}$	$= (2 \times 5\text{-g}) + (1 \times 10\text{-g})$	$= 20\text{g}$

25. Textbook, page 88, Solving Equations, questions 1 to 10

1. $17 = \square + 5$

$\square = 12$

Put 17 blocks in the left pan and 5 blocks in the right pan.



Add 12 blocks to the right pan to balance the scale.

2. $7 \times \square = 84$

$\square = 12$

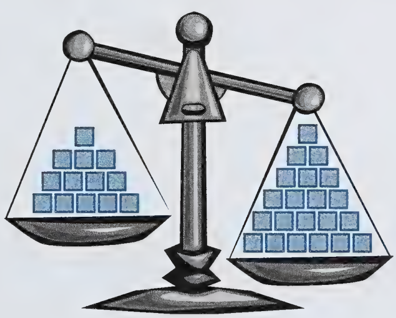
Work backwards. To find the number you must multiply by 7 to get 84, divide.

$84 \div 7 = 12$

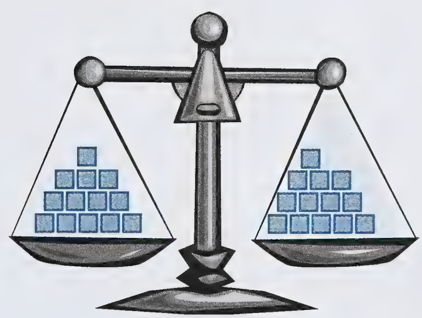
3. $13 = 21 - n$

$n = 8$

Put 13 blocks in the left pan and 21 blocks in the right pan.



Remove 8 blocks from the right pan to balance the scale.



4. $12 = t \div 12$

$t = 144$

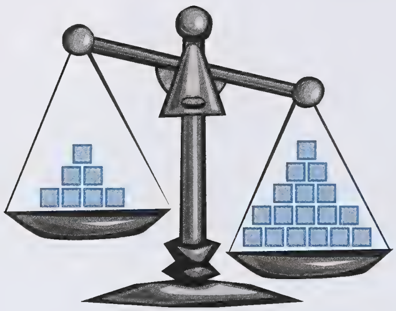
Work backwards. To find the number you must divide by 12 to get 12, multiply.

$12 \times 12 = 144$

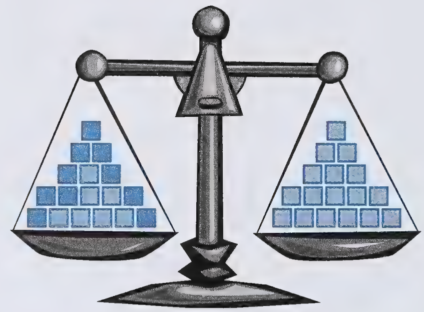
5. $7 + x = 8 + 9$

$x = 10$

Put 7 blocks in the left pan and $8 + 9 = 17$ blocks in the right pan.



Add 10 blocks to the left pan to balance the scale.



6. $25 - 9 = 2 \times \square$

$\square = 8$

Put $25 - 9 = 16$ blocks in the left pan.

Add two sets of 8 blocks each to the right pan to balance the scale.

7. $\square \times 6 = 84 \div 2$

$\square = 7$

Work backwards.

$84 \div 2 = 42$

To find the number you must multiply 6 by to get 42, divide.

$42 \div 6 = 7$

8. $n + 2 = 30 - 4$

$n = 24$

Work backwards.

$30 - 4 = 26$

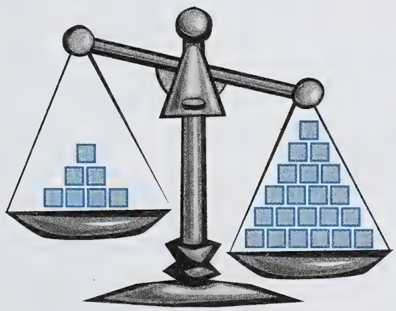
To find the number you must add to 2 to get 26, subtract.

$26 - 2 = 24$

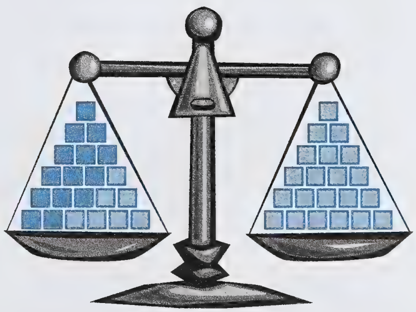
9. $d + 7 = 3 \times 7$

$d = 14$

Put 7 blocks in the left pan and $3 \times 7 = 21$ blocks in the right pan.



Add 14 blocks to the left pan to balance the scale.



10. $48 \div b = 24 \div 4$

$b = 8$

Work backwards.

$24 \div 4 = 6$

To find the number you must divide 48 by to get 6, divide.

$48 \div 6 = 8$

26. Textbook, page 87, On Your Own, question 1

1. There are 5 blue cubes and 8 red cubes in the bag. Solution strategies will vary. Sample answers are given.

- Start with 1 B and 4 R (3 more R than B). Continue adding an extra B and R until there is a total of 13 cubes.

B R R R R	B R B R R R R	B R B R B R R R R	B R B R B R B R R R R	B R B R B R B R B R R R R
5 cubes	7 cubes	9 cubes	11 cubes	13 cubes

- Using a table

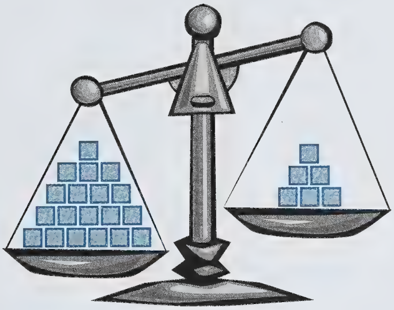
Blue Cubes	Red Cubes	Total Cubes
1	$1 + 3 = 4$	$1 + 4 = 5$
2	$2 + 3 = 5$	$2 + 5 = 7$
3	$3 + 3 = 6$	$3 + 6 = 9$
4	$4 + 3 = 7$	$4 + 7 = 11$
5	$5 + 3 = 8$	$5 + 8 = 13$

27. Textbook, page 89, Practise Your Skills, questions 1, 3, and 4

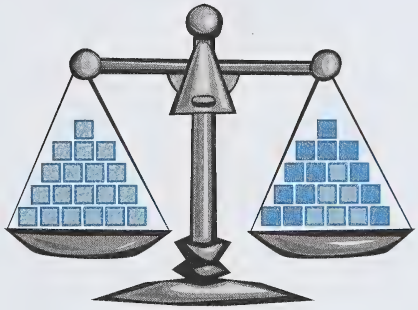
1. $19 = n + 6$

$n = 13$

Put 19 blocks in the left pan and 6 blocks in the right pan.



Add 13 blocks to the right pan to balance the scale.



2. $8 \times \square = 64$

$\square = 8$

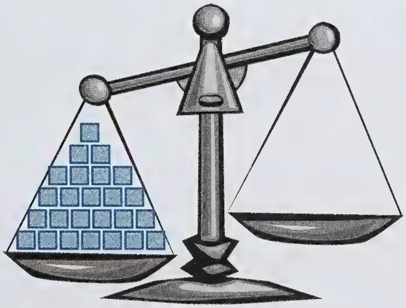
Work backwards. To find the number you must multiply 8 by to get 64, divide.

$64 \div 8 = 8$

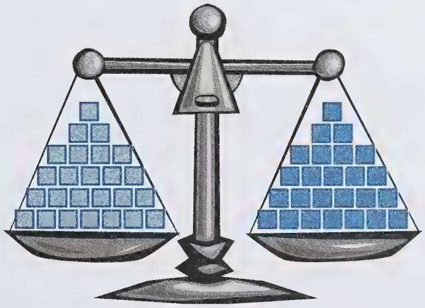
3. $34 - 9 = 5 \times n$

$n = 5$

Put $34 - 9 = 25$ blocks in the left pan.



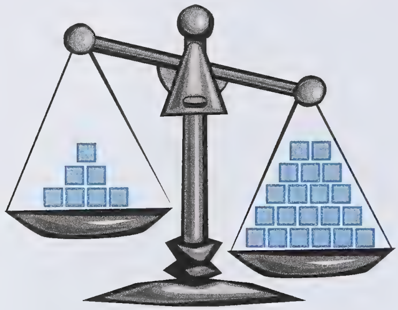
Add five sets of 5 blocks each to the right pan to balance the scale.



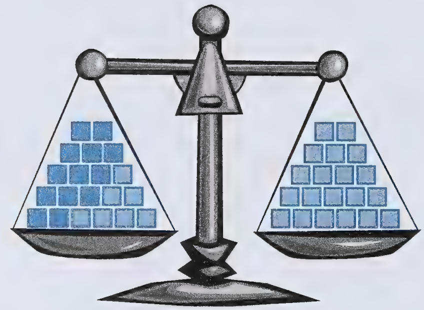
4. $d + 7 = 40 \div 2$

$d = 13$

Put 7 blocks in the left pan and $40 \div 2 = 20$ blocks in the right pan.



Add 13 blocks to the left pan to balance the scale.



5. $49 \div 7 = 14 \div \square$

$\square = 2$

Work backwards.

$49 \div 7 = 7$

To find the number you must divide by 14 by to get 7, divide.

$14 \div 7 = 2$

6. $51 - 6 = n \times 5$

$n = 9$

Work backwards.

$51 - 6 = 45$

To find the number you must multiply 5 by to get 45, divide.

$45 \div 5 = 9$

7. $16 = \square \div 2$

$\square = 32$

Work backwards. To find the number you must divide by 2 to get 16, multiply.

$2 \times 16 = 32$

8. $12 \times n = 49 - 1$

$n = 4$

Work backwards.

$49 - 1 = 48$

To find the number you must multiply 12 by to get 48, divide.

$48 \div 12 = 4$

28. Textbook, page 87, Practise Your Skills

Class Size	Clues	Number of Girls	Number of Boys
35	7 more boys than girls	14	21
30	2 times as many girls as boys	20	10
28	6 fewer boys than girls	17	11

Clue: 7 more boys than girls.
Use a guess-and-test strategy.

Number of Girls	Number of Boys	Class Size
10	17	27 (not enough)
12	19	31 (not enough)
14	21	35 (just right)

Clue: 2 times as many girls as boys
Use a guess-and-test strategy.

Number of Girls	Number of Boys	Class Size
24	12	36 (too many)
18	9	27 (not enough)
20	10	30 (just right)

Clue: 6 fewer boys than girls. Using a guess-and-test strategy.

Number of Girls	Number of Boys	Class Size
15	9	21 (not enough)
20	14	34 (too many)
17	11	28 (just right)

29. Textbook, pages 30 to 31, Problem Bank, questions 1 to 5

1. a.

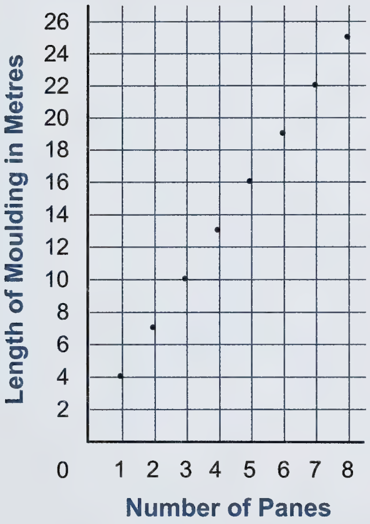
Window	Number of Panes	Length of Moulding in Metres
	1	4
	2	7
	3	10
	4	13
	5	16
	6	19
	7	22
	8	25

The following patterns can be seen:

- Each time a pane is added, 3 more metres of moulding are needed.
- The number of metres of moulding (m) is 1 more than 3 times the number of panes (p).

$$m = (3 \times p) + 1$$

b.



c. A window with 10 panes would have 31 m of moulding. 31 is 1 more than 3 times 10.

2. a. Seven cents GST is added for every dollar in the price. This means that the cost (c) including GST is 1.07 times the price (p).

$$c = 1.07 \times p$$

b. The price of an article is \$7.00 if its cost with GST is \$7.49.

Price	Cost with GST Included
\$5.00	\$5.35
\$6.00	\$6.42
\$7.00	\$7.49

c. An article priced at \$12.00 will cost \$12.84 after GST is added.

For each dollar, \$0.07 is added.

$12 \times \$0.07 = \$0.84$

$12 + \$0.84 = \$12.84$

3. a.

Number of Books	Total Mass (kg)
1	0.5
2	1
3	1.5
4	2
5	2.5
6	3
7	3.5
8	4
9	4.5
10	5

b. • The total mass (m) of the books is half the number of books (b).

$(m = b \div 2)$

• For each book that is added, the total mass increases by 0.5 kg.

• The mass of a package containing 15 books is $15 \div 2 = 7.5$ kg.

4. a. Multiply the first number by the second.
b. Add the first and second numbers; multiply the sum by the third number.
c. Divide the first number by the second.

5. a.

Quantity 1	Quantity 2
3	1
4	3
5	5
6	7
7	9
8	11
9	13

Multiply the number by 2, and then subtract 5.

b.

Quantity 1	Quantity 2
6	23
8	31
10	39
12	47
14	55
16	63
18	71

Multiply the number by 4, and then subtract 1.

c.

Quantity 1	Quantity 2
1	4
2	7
3	12
4	19
5	28
6	39
7	52

Multiply the number by itself, and then add 3.

Just the Facts

Multiplication and Division Facts

0	2	15	3	0
7	8	0	30	5
2	4	63	9	24
8	18	6	0	1
28	2	35	2	4

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Learning Aids

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100